Quiz 3:

1. (§10.4, # 19, 10 points) Determine if the series below converges or state that it diverges.

\[\sum_{j=2}^{\infty} \frac{1}{j \ln j}\]

Note \(j \ln j > 0\) for all \(j \geq 2\).

\[
\int_{2}^{\infty} \frac{1}{j \ln j} \, dj = \lim_{b \to \infty} \int_{2}^{b} \frac{1}{j \ln j} \, dj = \lim_{b \to \infty} \int_{\ln 2}^{\ln b} \frac{1}{u} \, du = \ln |u| \bigg|_{\ln 2}^{\ln b} = \ln b - \ln 2 = \ln \frac{b}{2} \to \infty.
\]

Since the improper integral diverges by the integral test, the series also diverges.

2. (§10.3, # 45, 10 points) Write the repeating decimal \(0.0\overline{9} = 0.09090909\ldots\) as a geometric series. Then determine the limit of the series.

\(0.0\overline{9} = 0.09 + 0.09\left(\frac{1}{100}\right) + 0.09\left(\frac{1}{100}\right)^2 + \ldots\)

\[\sum_{n=0}^{\infty} 0.09\left(\frac{1}{100}\right)^n\]

\[= \frac{0.09}{1 - \frac{1}{100}} = \frac{0.09}{\frac{99}{100}} = \frac{0.09 \times 100}{99} = \frac{9}{99} = \frac{1}{11}\]