MA 125 - Calculus I
Exam 2 : Part I
25 March 2010

Instructions:

• You have as long as you need to work on the first portion of this exam but it is written
to take a maximum of 20 minutes. When you finish, turn it in and only then you are
allowed to use your calculator for the remaining portion of this exam.

• You may not use any outside assistance on this exam. You may not use books,
notebooks, other people's exams, cell phones, mp3 players, or any other materials to
cheat on this exam.

• You may not use graphing calculator on this portion of the exam.

• If you are caught cheating on either portion of the exam, you will be given a 0 for
both portions.

• You must write clearly, give exact answers and fully reduce fractions to receive full
credit. Unreadable, approximate and unreduced answers will receive only partial
credit.

• You must show all your work to receive full credit unless otherwise stated.

Name: KEY
Score:______________ /100 Points
1. (5 points each) Determine the indicated derivatives.

(a) \( y' \) where \( y = x^2 + 3x \)
\[
2x + 3 = y' \quad \text{power rule}
\]

(b) \( df/dx \) where \( f(x) = x \sin x \)
\[
\frac{df}{dx} = \sin x + x \cos x \quad \text{product rule}
\]

(c) \( dg/dx \) where \( g(x) = \tan(x^2 + 1) \)
\[
\frac{dg}{dx} = 2x \sec^2(x^2 + 1) \quad \text{chain rule}
\]

(d) \( k'(x) \) where \( k(x) = \frac{x^4 - 8x}{2x + 1} \)
\[
k'(x) = \frac{4x^3 - 8(2x+1) - 2(x^4 - 8x)}{(2x+1)^2} \quad \text{quotient rule}
\]

(e) \( d^3l/dx^3 \) where \( l(x) = 3x^2 + 8x \)
\[
\frac{d^3l}{dx^3} = 0 \quad \text{power rule}
\]
\[
\frac{dl}{dx} = 6x + 8 \quad \text{constant}
\]

(f) \( dy/dx \) where \( x = 5t - 1 \) and \( y = t^2 - 3 \)
\[
\frac{dy}{dx} = \frac{2t}{5} \quad \text{eliminate thets.}
\]
\[
x = \frac{5t - 1}{5} \quad y = \left( \frac{t^2}{5} \right) - 3
\]
\[
\frac{x + 1}{5} = t \quad \frac{2u}{5} = 2u \cdot \frac{1}{5}
\]
\[
\frac{x + 1}{5} = \frac{2u}{5} \quad \frac{2}{u} = \frac{2(u + 1)}{5}
\]
2. (5 points) Using only the definition of the derivative, find \( f'(x) \) if \( f(x) = x^2 - 3x \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}
\]

\[
= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 3x - 3h - x^2 + 3x}{h}
\]

\[
= \lim_{h \to 0} \frac{2hx + h^2 - 3h}{h}
\]

\[
= \lim_{h \to 0} 2x + h - 3 = 2x - 3
\]

3. (4 points) Consider the parametric equations and parameter interval below.

\[ x = 3 \cos t, \quad y = 3 \sin t, \quad -\pi/2 \leq t \leq \pi/2 \]

Describe using full sentences what portion of the graph for the associated Cartesian equation \( x^2 + y^2 = 9 \) is traced out by the parametric equations on the given interval. Also describe whether the tracing was done in a clockwise or counterclockwise direction and justify both of your answers.

\[
\frac{x}{3} = \cos t \quad \left( \frac{y}{3} \right)^2 + \left( \frac{x}{3} \right)^2 = 1
\]

\[
\frac{y}{3} = \sin t \quad \frac{y^2}{9} + \frac{x^2}{9} = 1
\]

\[
y^2 + x^2 = 9 \quad \text{Ending at } \quad t = \pi/2
\]

\[
x = 3 \cos (-\pi/2) = 0 \quad \text{at } \quad t = 0 \quad y = 3 \sin (\pi/2) = 3
\]

\[
y = 3 \sin (-\pi/2) = -3 \quad \text{at } \quad t = \pi/2 \quad (0,3)
\]

\[
\text{The portion of the graph traced out is a half of a circle with a radius of } \sqrt{3}, \text{ that starts at } \quad t = -\pi/2 \quad \text{and stops at } \quad t = \pi/2 \quad (for \text{ the given interval.) } \text{The tracing was done in a counterclockwise direction} \text{ because as } t \text{ increased so did the output values.}
MA 125 - Calculus I
Exam 2: Part II

Instructions:

- You have the remainder of the period to finish portion II of this exam.
- You may use your graphing calculator on this portion of the exam after you have turned in portion I. All other instructions from Portion I apply to this portion of the exam. Show all your work to receive full credit unless otherwise stated.

1. (5 points each) Determine the equation of the tangent line to the curve described by the function \( y = \sec x \) at the indicated point \((4\pi/3, -2)\).

   \[
   \begin{align*}
   \frac{\text{Find } y'}{y'} &= \sec x \tan x \\
   m &= \sec \left(\frac{4\pi}{3}\right) \tan \left(\frac{4\pi}{3}\right) \\
   m &= -2 \sqrt{3} \\
\end{align*}
   \]

   \[y + 2 = -2\sqrt{3} \left(x - \frac{4\pi}{3}\right)\]

2. (4 points) Find a parametrization for the line segment with endpoints \((3, -4)\) and \((7, 2)\).

   \[
   \begin{align*}
   \text{Equations of the line} & \quad x = 3 + 4t \\
   & \quad y = -4 + 6t \\
\end{align*}
   \]

   When \( t = 0 \),
   \[
   \begin{align*}
   7 &= 3 + 4a \quad \rightarrow \quad a = 1 \\
   2 &= -4 + 6b \quad \rightarrow \quad b = 6 \\
\end{align*}
   \]

   Plug \( a, b \) into original equations
   \[
   \begin{align*}
   x &= 3 + 4t \\
   y &= -4 + 6t \\
\end{align*}
   \]

   \( x = 3 + 4t \) and \( y = -4 + 6t \) are the parametric equations and \( 0 \leq t \leq 1 \) is the interval for parametrization.
3. (3 points each) A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height \( s(t) = 24t - 0.8t^2 \) meters in \( t \) seconds.

(a) Find an equation to describe the rock's velocity at time \( t \).

\[
3(t) = 24t - 0.8t^2
\]

\[
3'(t) = v(t) = \frac{d}{dt} \left( 24t - 0.8t^2 \right)
\]

\[
v(t) = \frac{d}{dt} 24t - \frac{d}{dt} 0.8t^2
\]

\[
v(t) = 24 - 1.6t
\]

(b) Determine at exactly what time the rock stops rising and begins to fall.

When velocity \( = 0 \), the rock will stop rising.

Therefore,

\( 0 = 24 - 1.6t \)

\(-24 = -1.6t \)

\( t = 15 \) sec

After 15 secs, the rock stops rising.

(c) Determine exactly how high the rock goes before it begins to fall.

\[
S_{\text{max}} = s(15) = 24(15) - 0.8(15)^2
\]

\[
= 180 \text{ meters}
\]

(d) Find an equation to describe the rock's acceleration at time \( t \).

\[
v(t) = 24 - 1.6t
\]

\[
v'(t) = a(t) = \frac{d}{dt} (24 - 1.6t)
\]

\[
a(t) = \frac{d}{dt} 24 - \frac{d}{dt} 1.6t
\]

\[
a(t) = 0 - 1.6
\]

\[
a(t) = -1.6
\]
4. (3 points each) Determine if each of the statements is either true or false. You must write the entire word TRUE or FALSE to receive full credit. If you write only T or F, you will receive at most one point. You do not have to show your work for this problem.

(a) The Cartesian equation \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) of an ellipse is parametrized by the equations:

\[
x = a \cos t, \quad y = b \sin t, \quad 0 \leq t \leq 2\pi.
\]

\[
\frac{d}{dt} \cos t = -\sin t \quad \frac{d}{dt} \frac{x}{a} = \frac{x}{a}
\]

\( \text{True} \)

(b) Every polynomial is differentiable on its entire domain \((-\infty, \infty)\).

\( \text{True} \)

(c) \(\frac{d^{100}}{dx^{100}} \cos x = \sin x\)

\(249 \approx 3\)

\( \text{True} \)

(d) The third derivative of velocity with respect to time is jerk.

\[
\begin{align*}
\gamma' &= \text{velocity} \\
\gamma'' &= \text{accel} \\
\gamma''' &= \text{jerk}
\end{align*}
\]

\( \text{False} \)

(e) Given a parametrized curve, if all three derivatives exist and \( dy/dt \neq 0 \) then:

\[
\frac{dy}{dx} = \frac{dx/dt}{dy/dt}
\]

\( \text{False} \)
5. Consider the function \( f(x) = x^{2/5} \).

(a) (4 points) Graph the function \( f(x) \) on a Cartesian coordinate system with 
\( x\)-window \([-2, 2]\), \( y\)-window \([-2, 2]\) and both scales 0.5. (Hint: You do not need to show your work for (5a) so use your calculator for assistance.)

(b) (3 points) From your graph in (5a), at what \( x \) value does it appear \( f(x) \) could have a vertical tangent line?

\[ X = 0 \]

(c) (3 points) Show that in fact at the \( x \) value found in (5b), the graph of \( f(x) \) does not have a vertical tangent line.

\[ f(x) = x^{2/5} \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h)^{2/5} - f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{(x+h)^{2/5} - x^{2/5}}{h} \]

\[ = \lim_{h \to 0} \frac{h}{h^{2/5}} \]

This allows there to be positive and negative slopes, since the limit will be different as it approaches 0 from the left and right there is no vertical tangent line.
6. (4 points) A pebble is dropped in a pond to cause ripples of cocentric circles. The area enclosed by the outer most ring of ripples is \( A = \pi r^2 \) and changes with the radius. At what rate \( \frac{dA}{dt} \) does the area change with respect to the radius when \( r = 3 \)?

\[
A' = 2\pi r \\
= 2\pi (3) = 6\pi
\]

7. Consider the function: \( g(x) = \begin{cases} 
  x + c & x < 0 \\
  \cos x & x \geq 0
\end{cases} \)

(a) (4 points) Is there a value of \( c \) that will make the function \( g(x) \) continuous at \( x = 0 \)?

\[
\lim_{x \to 0^-} x + c = c \\
\lim_{x \to 0^+} \cos x = 1
\]

If both one sided limits are equal, \( g(x) \) is continuous. So, \( c = 1 \) makes it continuous at \( x = 0 \).

(b) (3 points) For the value of \( c \) found in (7a), determine if the function \( g(x) \) is differentiable at \( x = 0 \)? Explain your reasoning.

As \( x \to 0 \) from the left, the derivative is \( \frac{d}{dx} x = 1 \)

As \( x \to 0 \) from the right, the derivative is \( \frac{d}{dx} \cos x = -\sin x \)

The left and right derivatives do not agree. Therefore, \( g(x) \) is not differentiable at \( x = 0 \).