MA 125 - Calculus I
Exam 1
18 February 2010

Instructions:

• You may not use any outside assistance on this exam. You may not use books, notebooks, other people’s exams, cell phones, mp3 players, or any other materials to cheat on this exam.

• You may not use a graphing calculator on this exam.

• If you are caught cheating on any question of the exam, your grade for the entire exam will be a 0.

• You must write clearly, give exact answers and fully reduce fractions to receive full credit. Unreadable, approximate and unreduced answers will receive only partial credit.

• You must show all your work to receive full credit unless otherwise stated.

Name:______________________________
Score:______________ /100 Points
1. (5 points each) Evaluate the following limits. If a limit does not exist, explain why not.

(a) \( \lim_{x \to 5} \frac{x-5}{x^2-25} \)

(b) \( \lim_{t \to 0} \frac{\sin 3t}{5t} \)

(c) \( \lim_{x \to 2} \sqrt{4-x^2} \)

(d) \( \lim_{h \to 0} \frac{\sqrt{h^2+9}-3}{h} \)
2. (5 points each) Consider the equation $g(x) = x^2 - 2x - 3$.

   (a) What is the average rate of change of $g(x)$ on the interval $[0, 5]$?

   (b) What is the equation of the secant line connecting $g(0)$ and $g(5)$?

   (c) What is the slope of the curve $g(x)$ at the point $(3, 0)$?
3. (5 points each) Consider the following the function.

\[ l(x) = \begin{cases} 
-x & x < 0 \\
1 + x & 0 \leq x < 2 \\
2 & x = 2 \\
0 & x > 2 
\end{cases} \]

(a) Graph the function \( l(x) \) on a Cartesian coordinate system being sure to label and scale all axis.

(b) At what values \( x = c \) does the function \( l(x) \) fail to have a two sided limit? At these values \( c \), determine the left hand limit, \( \lim_{x \to c^-} l(x) \).
4. (10 points) Determine the equations of all asymptotes for the function below.

\[ c(x) = \frac{x^2 - 5x + 4}{x^2 - 1} \]

5. (10 points) For the function below, determine the continuous extension. Be sure to justify your answers.

\[ f(x) = \frac{x(x - 4)}{x^2 - 16} \]
6. (3 points each) Determine if each of the statements is either true or false. You must write the entire word TRUE or FALSE to receive full credit. If you write only T or F, you will receive at most one point. You do not have to show your work for this problem.

(a) \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) = 0 \)

(b) The greatest integer function or floor function, \( f(x) = \lfloor x \rfloor \), is discontinuous at all integer values of \( x \).

(c) The rational function \( j(x) = \frac{2x^2 + 3}{x - 4} \) has an oblique asymptote at \( y = 2x + 8 \).

(d) If \( \lim_{x \to a^+} t(x) = -\infty \) then \( x = a \) is a vertical asymptote to the graph \( t(x) \).

(e) If the product function \( h(x) = f(x) \cdot g(x) \) is continuous at \( x = c \) then \( f(x) \) and \( g(x) \) must both be continuous at \( x = c \).
7. (5 points) If \( \cos x + 3 \leq h(x) \leq 3x^2 + 4 \), determine

\[
\lim_{x \to 0} h(x).
\]

8. (5 points) Using the Intermediate Value Theorem, show that the function below has a root between \(-1\) and \(2\).

\[
r(x) = x^3 - x - 1
\]
9. (5 points each) Determine where the following functions are continuous.

(a) \( s(x) = \sqrt{\frac{1}{x-5}} \)

(b) \( t(\theta) = \tan \theta + \cot \theta \)
10. Extra Credit: (5 points) Consider the function \( q(x) = \sqrt{x - 7}, \ x_0 = 23, \) and \( \epsilon = 1. \) Find a number \( \delta > 0 \) such that for all \( x: \)

\[
0 < |x - x_0| < \delta \implies |q(x) - L| < \epsilon
\]

where \( L \) is the limit of \( q(x) \) when \( x \) approaches \( x_0. \)