Results from the 2012 AP Calculus AB and BC Exams

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Outline

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3. The Reading
   (a) Leadership
   (b) The Flow and Question Teams
   (c) Logistics and Numbers
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AP Calculus Development Committee

- Co-Chair: Thomas Becvar, St. Louis University High School, St. Louis, MO
- Co-Chair: Tara Smith, University of Cincinnati, Cincinnati, OH
- Vicki Carter, West Florence High School, Florence, SC
- Donald King, Northeastern University, Boston, MA
- James Sellers, Penn State University, State College, PA
- Jennifer Wexler, New Trier High School, Winnetka, IL
- Chief Reader: Stephen Kokoska, Bloomsburg, University, PA
- College Board Advisor: Kathleen Goto, Iolani School, Honolulu, HI
- ETS Consultants: Fred Kluempen, Craig Wright, Chad Griep

APDC

Committee Responsibilities

- Plan, develop, and approve each exam.
- Participate in the Reading.
- Consider statistical analysis associated with the exam.
- Review, revise, and update the Course Description.
- Advise the Chief Reader on Reading issues.
- Participate in Outreach Events.
- Four meetings each year. Additional virtual meetings.

Exams

AP Calculus Exams

- US Main: United States, Canada, Puerto Rico, US Virgin Islands
- Form A: US Alternate Exam: late test
- Form I: International Main Exam
- Form J: International Alternate Exam

Parts

- Section I: Multiple Choice. Section II: Free Response.
- Calculator and Non-Calculator Sections
- AB and BC Exams.

The Reading Leadership

- Chief Reader: Stephen Kokoska
  Bloomsburg University, PA
- Assistant to the Chief Reader (ACR): Tom Becvar
  St. Louis University High School, MO
- Chief Reader Associate (CRA): Stephen Davis
  Davidson College, NC
- Chief Aide: Craig Turner
  Georgia College and State University, GA
- AB Exam Leader: Jim Hartman
  College of Wooster, OH
- BC Exam Leader: Peter Atlas
  Concord Carlisle Regional High School, MA

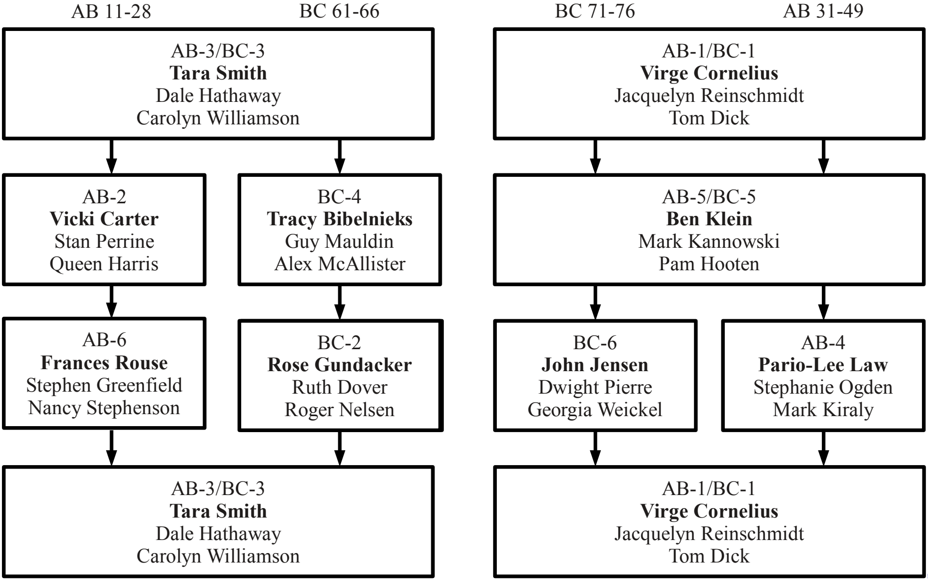
Question Leaders

- AB-1/BC-1: Water in a Tub
  Virge Cornelius, Lafayette High School, MS
- AB-2: Area, Volume
  Vicki Carter, West Florence High School, SC
- AB-3/BC-3: Fundamental Theorem of Calculus
  Tara Smith, University of Cincinnati, OH
- AB-4: Piecewise Function
  Pario-Lee Law, D'Evelyn Junior Senior HS, CO
- AB-5/BC-5: Baby Bird
  Ben Klein, Davidson College, NC
- AB-6: Particle Motion
  Frances Rouse, UMS - Wright Preparatory School, AL

Question Leaders

- BC-2: Particle Motion
  Rose Gundacker, University of St. Thomas, MN
- BC-4: Approximation
  Tracy Bibelnieks, Augsburg College, MN
- BC-6: Infinite Series
  John Jensen, Rio Salado College, AZ
- Alternate Exam:
  Julie Clark, Hollins College, VA
  Kory Swart, Kirkwood Community College, IA
- International Exam:
  Deanna Caveny, College of Charleston, SC
  Chris Lane, Pacific University, OR

The Reading: 2012 Grading Flow



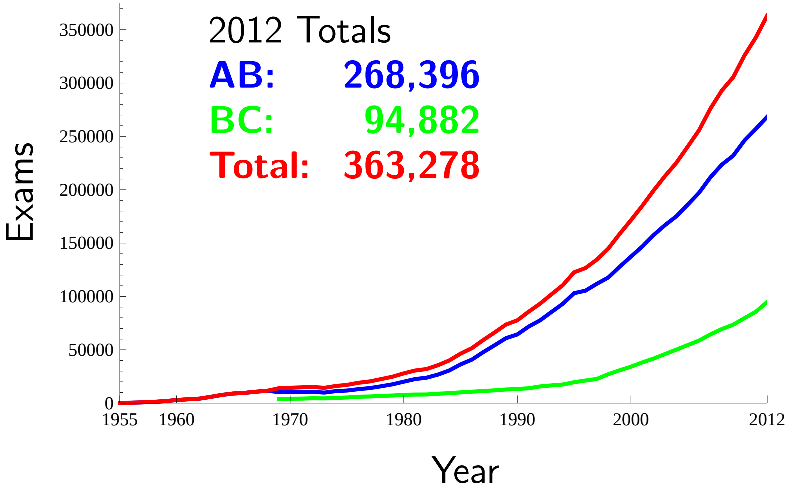
The Reading

Logistics and Participants

- Kansas City:
  Convention Center (versus college campus)
  Westin Hotel (versus college dorms)
- Total Participants: 853
- High School: 55%   College: 45%
- 50 states, DC, and other countries

The Reading

Number of (all) AP Calculus Exams



The Reading

2012 Scores (US Main Exam)

US Main			
Score	AB	BC	AB subscore
5	24.9%	50.6%	60.3%
4	17.0%	16.2%	16.7%
3	17.4%	16.2%	9.0%
2	10.3%	5.4%	5.8%
1	30.5%	11.8%	8.2%

The Reading

General Comments

- We awarded points for good calculus work, if the student conveyed an understanding of the appropriate calculus concept.
- Students must show their work (bald answers).
- Students must communicate effectively, explain their reasoning, and present results in clear, concise, proper mathematical notation.
- Practice in justifying conclusions using calculus arguments.
- Decimal presentation errors, use of intermediate values.

2012 Free Response

General Information

- Six questions on each exam (AB, BC).
- Three common questions: AB-1/BC-1, AB-3/BC-3, AB-5/BC-5.
- Scoring: 9 points for each question.
- Complete and correct answers earn all 9 points.
- The scoring standard is used to assign partial credit.

2012 Free Response: AB-1/BC-1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2012 Free Response: AB-1/BC-1

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$
$= 1.017 \text{ (or } 1.016)$

The water temperature is increasing at a rate of approximately $1.017\,°\text{F}$ per minute at time $t = 12$ minutes.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by $16\,°\text{F}$ over the interval from $t = 0$ to $t = 20$ minutes.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

(c)
$$\begin{aligned} \frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ &= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ &= \frac{1}{20} \cdot 1215.8 = 60.79 \end{aligned}$$

This approximation is an underestimate since a left Riemann sum is used and the function W is strictly increasing.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

(d)
$$\begin{aligned} W(25) &= 71.0 + \int_{20}^{25} W'(t)\,dt \\ &= 71.0 + 2.043155 = 73.043 \end{aligned}$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2012 Free Response: AB-1/BC-1

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	3.96	2.88	5.6	16.6
BC	5.88	2.55	14.7	4.1

- In general, students performed well.
  Multiple entry points. No surprises.
- Part (a): Most students set up a difference quotient.
  Interpretation tougher. Needed correct units.
  Average rate of change (your answer).
- Part (b): Good job recognizing the FTC.
  Interpretation: change, units, and interval.

2012 Free Response: AB-1/BC-1

Results

- Part (c): Left Riemann sum and computation good. Explanation: inadequate or incorrect reasons.
- Part (d): Students did fairly well.

2012 Free Response: AB-1/BC-1

Common Errors

- Interpretation of the answer in the context of the problem.
- Correct units.
- Part (b): The meaning of the definite integral.
- Part (c): Incorrectly assumed the width of each subinterval was the same.

  Explanation associated with an underestimate.
- Part (d): Use of 0 as a lower bound on the definite integral.

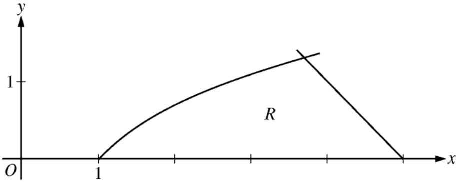
2012 Free Response: AB-1/BC-1

To Help Students Improve Performance

- In general, most students were able to apply appropriate concepts and compute correct numerical answers.
- Interpretation and communication of results.
- Clearly indicate the mathematical steps to a final solution.

2012 Free Response: AB-2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

2012 Free Response: AB-2

$\ln x = 5 - x \Rightarrow x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $\text{Area} = \int_0^B \left(5 - y - e^y\right) dy$

$= 2.986 \text{ (or } 2.985)$

OR

$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x)\, dx$

$= 2.986 \text{ (or } 2.985)$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$

$3: \begin{cases} 2: \text{integrands} \\ 1: \text{expression for total volume} \end{cases}$

(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{equation} \end{cases}$

2012 Free Response: AB-2

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	3.09	3.10	6.9	41.0

- Area / volume problem with two regions: difficult for students.
- Working in x: OK in parts (a) and (b).
- Part (c) was very challenging for students working with respect to x.
- Part (a): Common solution in terms of x, 2 regions. For those in y: OK if correctly found $x = e^y$.

2012 Free Response: AB-2

Results

- Part (b): Students did well.
  Two distinct, separate integrals to find total volume.
- Part (c): Those working in terms of y more successful.
  Some complicated yet correct solutions in terms of x.

2012 Free Response: AB-2

Common Errors

- No calculator use to find (A, B).
  Point of intersection reported as $(4, 1)$.
- Solving for x in terms of y (inverse functions).
- Part (a): $\displaystyle\int_1^5 (5 - x - \ln x)\, dx$
- Incorrect limits:
  0 as a lower bound, 4 as an upper bound.
- Part (b): $\displaystyle\int_1^5 (5 - x - \ln x)^2\, dx$ (constant π)
- Part(c): Equation in terms of x.

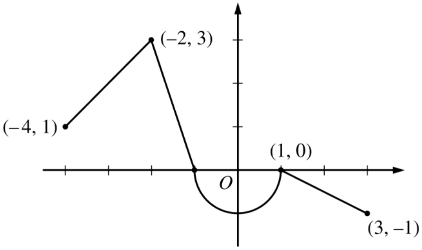
2012 Free Response: AB-2

To Help Students Improve Performance

- Practice in solving for x in terms of y (inverse functions).
- Computing areas in which there are distinct right and left boundaries.
- Using the best, most efficient method for finding the area of a region.
- Computing the volume of a solid with known cross sectional area.
- Computing the area of certain known geometric regions at an arbitrary point x.

2012 Free Response: AB-3/BC-3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

2012 Free Response: AB-3/BC-3

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$$

$$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$$

$2: \begin{cases} 1: g(2) \\ 1: g(-2) \end{cases}$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$

$g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$2: \begin{cases} 1: g'(-3) \\ 1: g''(-3) \end{cases}$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: x = -1 \text{ and } x = 1 \\ 1: \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ since $g''(x) = f'(x)$ changes sign at each of these values.

$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$

2012 Free Response: AB-3/BC-3

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	2.67	2.58	1.3	30.9
BC	4.29	2.61	3.8	11.3

- Part (a): Inability to evaluate a definite integral in terms of area.
- Part (b): Correct connections,
  $g'(x) = f(x)$ and $g''(x) = f'(x)$.
  Fundamental Theorem of Calculus errors.

2012 Free Response: AB-3/BC-3

Results

- Part (c): Correctly identified locations of horizontal tangent lines.
  Classification of these points: good.
  Justifications lacking.
- Part (d): Confusion about when a function has an inflection point.

2012 Free Response: AB-3/BC-3

Common Errors

- Part (a): 3 as an upper bound.
  Area of a semicircle.
  Add / subtract areas.
  Evaluation of a definite integral: $f(b) - f(a)$
- Part (b): Sign errors, $-2 - (-4)$
  FTC: $g'(x) = f(x) - f(1)$ and $g''(x) = f'(x) - f'(1)$
  Use of 3 rather than -3.

2012 Free Response: AB-3/BC-3

Common Errors

- Part (c): Insufficient communication, $g'(x) = 0$.
  Most students did find $x = -1$ and $x = 1$.
  Inappropriate notation in reporting $x = \pm 1$
  $x = -1$ as the location of a relative minimum.
  Use of ordered pairs on the graph of f when discussing points on the graph of g.

- Part (d): Included all points were f' was undefined $(x = -1)$
  Included endpoints, $x = -4$ and $x = 3$.
  Ambiguous justifications: the derivative, the slope.
  No reference to the proper function: f, g, f', or g'.

2012 Free Response: AB-3/BC-3

To Help Students Improve Performance

- Emphasize interpretation of first and second derivatives.
- The ability to read information from graphs.
- Evaluating definite integrals from a graph.
- Finding and classifying critical points, and points of inflection.
- The Fundamental Theorem of Calculus.
- Use of mathematical notation, absence of ambiguity.

2012 Free Response: AB-4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$	$2 :\ f'(x)$
(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$ $f(-3) = \sqrt{25 - 9} = 4$ An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.	$2 : \begin{cases} 1 : f'(-3) \\ 1 : \text{answer} \end{cases}$

2012 Free Response: AB-4

(c) $\lim\limits_{x\to -3^-} g(x) = \lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^-} \sqrt{25 - x^2} = 4$

$\lim\limits_{x\to -3^+} g(x) = \lim\limits_{x\to -3^+} (x+7) = 4$

Therefore, $\lim\limits_{x\to -3} g(x) = 4.$

$g(-3) = f(-3) = 4$

So, $\lim\limits_{x\to -3} g(x) = g(-3).$

Therefore, g is continuous at $x = -3$.

$$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$$

(d) Let $u = 25 - x^2 \;\Rightarrow\; du = -2x\,dx$

$$\int_0^5 x\sqrt{25 - x^2}\,dx = -\frac{1}{2}\int_{25}^{0} \sqrt{u}\,du$$

$$= \left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$$

$$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$$

$$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

2012 Free Response: AB-4

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	4.09	2.61	2.8	14.3

- Most students were able to enter this problem in a variety of ways in order to earn points.
- Students did well in parts (a) and (b).
- Part (c): Most did not correctly use the definition of continuity.
- Part (d): All or nothing, depending on antidifferentiation.

2012 Free Response: AB-4

Common Errors

- Part (a): Chain rule error or sign error.
- Part (b): Arithmetic errors.
  Use of $x = 3$ instead of $x = -3$.
  Evaluation of $-x$ when $x = -3$.

2012 Free Response: AB-4

Common Errors

- Part (c): No appeal to the definition of continuity.
  Checked $f(x)$ and $x+7$ at $x=-3$.
  Poor notation and communication.
  Limit symbol without an argument: $\lim_{x \to -3} = 4$
  Did not consider the value $g(-3)$.
  Found $f(-3)$ but never stated that $g(-3) = f(-3)$.
- Part (d): Basic misunderstanding of how to integrate a product of functions.
  Separation of factors, integrate each separately.
  Some sign and arithmetic errors.

2012 Free Response: AB-4

To Help Students Improve Performance

- Standard mathematical notation is very important in order to communicate solution steps.
  The proper use of parentheses.
- Definitions and theorems are important in the study of calculus.
  Justifications based on these results.
- Simplification of results not necessary on the AP Calculus Exam.
  (They are necessary in my class.)
  Frequent arithmetic, evaluation, and algebraic errors.
  (The student has said too much.)

2012 Free Response: AB-5/BC-5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

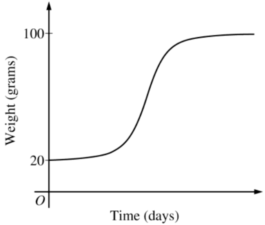
$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



2012 Free Response: AB-5/BC-5

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Since $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$ the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

2012 Free Response: AB-5/BC-5

(c) $\dfrac{dB}{dt} = \dfrac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$$

$$-\ln\left|100 - B\right| = \frac{1}{5}t + C$$

Since $20 \le B < 100$, $\left|100 - B\right| = 100 - B$.

$$-\ln(100 - 20) = \frac{1}{5}(0) + C \;\Rightarrow\; -\ln(80) = C$$

$$100 - B = 80e^{-t/5}$$

$$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$$

$$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } B \end{cases}$$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

2012 Free Response: AB-5/BC-5

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	2.87	2.24	1.4	16.3
BC	4.75	2.55	7.1	5.5

- Part (a): Most students earned 2 points, used the method that appears on the standard.
- Part (b): Many students did not use the chain rule.
- Part (c): Separate the variables!

2012 Free Response: AB-5/BC-5

Common Errors

- Part (a): Did not make it sufficiently clear they were using the given differential equation.
  Poor communication.

- Part (b): No chain rule.

  Differentiating $\dfrac{dB}{dt}$ with respect to B.

  Answer left as $-\dfrac{1}{5}\,\dfrac{dB}{dt}$

- Part (c): Method of separation of variables.

2012 Free Response: AB-5/BC-5

To Help Students Improve Performance

- Practice in solving separable differential equations.
- Practice in explaining and interpretation of results.
- Students still need expert algebra skills.
- Communication skills.

2012 Free Response: AB-6

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \;\Rightarrow\; t = 3,\ 9$ The particle is moving to the left when $v(t) < 0$. This occurs when $3 < t < 9$.	$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$
(b) $\int_0^6 \lvert v(t) \rvert \, dt$	1 : answer

2012 Free Response: AB-6

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time $t = 4$ since velocity and acceleration have the same sign.

$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$

$$\begin{aligned} &= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4 \\ &= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right] \\ &= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

2012 Free Response: AB-6

Results

Exam	Mean	St Dev	% 9s	% 0s
AB	3.59	2.82	5.5	19.1

- Very few blank papers. Most earned points somewhere.
- Parts (a) and (b): Students did very well.
- Part (c): Most found $a(t)$ correctly.
- Part (d): Many attempted to find a solution using an alternate method.

2012 Free Response: AB-6

Common Errors

- Part (a): Incorrect intervals, left endpoint of 4. Use of an antiderivative of $v(t)$, instead of $v(t)$. Sign chart rather than written communication.

- Part (b): Integral for displacement: $\displaystyle\int_0^6 v(t)\,dt$ Initial condition included.

2012 Free Response: AB-6

Common Errors

- Part (c): Chain rule used incorrectly.
  Did not consider $v(4)$ (as well as $a(4)$).
  Sign errors in finding $v(4)$.
  Incorrect values for the two trigonometric functions at $t = 4$.
- Part (d): Initial condition not used.
  Incorrect antiderivative of $\cos\left(\frac{\pi}{6}t\right)$

2012 Free Response: AB-6

To Help Students Improve Performance

- Learn and be able to use the correct unit circle values.
- Communication skills. Proper mathematical notation.
- More practice with particle motion problems.
- Understand the difference between displacement and total distance traveled.
- Difference between speed and velocity.
- Initial value problems, especially cases in which the initial value is not 0 (or at 0).

2012 Free Response: BC-2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\dfrac{dx}{dt}\right|_{t=2} = \dfrac{2}{e^2}$

Since $\left.\dfrac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055 \text{ (or } 3.054)$$

$$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \dfrac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$$

2012 Free Response: BC-2

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

$$\begin{aligned} \text{Acceleration} &= \langle x''(4), y''(4) \rangle \\ &= \langle -0.041, 0.989 \rangle \end{aligned}$$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2012 Free Response: BC-2

Results

Exam	Mean	St Dev	% 9s	% 0s
BC	5.07	2.66	10.7	6.0

- Student performance generally very good. Understanding of parametrically defined curve.
- Arithmetic, algebra, decimal presentation errors.
- Correct presentations followed by incorrect answers (from calculator).
- Inappropriate use of initial condition.

2012 Free Response: BC-2

Common Errors

- Part (a): Horizontal movement using $\dfrac{dy}{dx}$ or $x(2)$.

  Arithmetic, algebra errors in computing the slope.

- Part (b): Initial condition not used.

  Symbolic antiderivative of $\dfrac{dx}{dt}$

  (abandoned, restart with calculator)

2012 Free Response: BC-2

Common Errors

- Part (c): Incorrect formula for speed.

  Derivatives of $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ analytically.

  OK, but quotient rule or algebra errors.

- Part (d): Use of $\left|\dfrac{dy}{dx}\right|$ as the integrand in computing distance.

  Use of the formula for arc length assuming $y = f(x)$.

2012 Free Response: BC-2

To Help Students Improve Performance

- Decimal presentation instructions and intermediate values.
- More practice with the use of the Fundamental Theorem of Calculus.

$$x(b) = x(a) + \int_a^b x'(t)\, dt$$

- Concepts of speed and total distance traveled.

2012 Free Response: BC-4

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f¢ the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$. to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

2012 Free Response: BC-4

(a) $f(1) = 15, \quad f'(1) = 8$

An equation for the tangent line is
$y = 15 + 8(x-1)$.

$f(1.4) \approx 15 + 8(1.4-1) = 18.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\int_1^{1.4} f'(x)\,dx \approx (0.2)(10) + (0.2)(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$

$f(1.4) \approx 15 + 4.6 = 19.6$

$3 : \begin{cases} 1 : \text{midpoint Riemann sum} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $f(1.2) \approx f(1) + (0.2)(8) = 16.6$

$f(1.4) \approx 16.6 + (0.2)(12) = 19.0$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(d) $T_2(x) = 15 + 8(x-1) + \frac{20}{2!}(x-1)^2$
$= 15 + 8(x-1) + 10(x-1)^2$

$f(1.4) \approx 15 + 8(1.4-1) + 10(1.4-1)^2 = 19.8$

$2 : \begin{cases} 1 : \text{Taylor polynomial} \\ 1 : \text{approximation} \end{cases}$

2012 Free Response: BC-4

Results

Exam	Mean	St Dev	% 9s	% 0s
BC	5.43	2.84	18.0	7.6

- In general, students did well.
- Midpoint Riemann sum and Taylor polynomial keys to success on this problem.
- Minimal supporting work.
- Incorrect values from the table or problem statement.

2012 Free Response: BC-4

Common Errors

- Part (a): Use of incorrect values,

  $f(1) = 8$, $f'(1) = 20$.

- Part (b): Trapezoidal sum.
  Use of four subintervals over $[1, 1.4]$
  Incorrect use of the Fundamental Theorem of Calculus.

2012 Free Response: BC-4

Common Errors

- Part (c): Arithmetic errors,

  $f'(x) \cdot \Delta x$ and/or $f(x) + f'(x) \cdot \Delta x$

  Unlabeled or poorly communicated work.

- Part (d): Did not use values from the table.
  Did not center the polynomial at $x = 1$.

2012 Free Response: BC-4

To Help Students Improve Performance

- Practice with problems involving tabular data.
- Clearly label values and answers.
- Practice with problems in context.
- Present distinct, labeled answers to each part of a question.

2012 Free Response: BC-6

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

2012 Free Response: BC-6

(a) $\left|\dfrac{x^{2n+3}}{2n+5}\cdot\dfrac{2n+3}{x^{2n+1}}\right| = \left(\dfrac{2n+3}{2n+5}\right)\cdot x^2$

$$\lim_{n\to\infty}\left(\frac{2n+3}{2n+5}\right)\cdot x^2 = x^2$$

$x^2 < 1 \;\Rightarrow\; -1 < x < 1$

The series converges when $-1 < x < 1$.

When $x = -1$, the series is $-\dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \cdots$

This series converges by the Alternating Series Test.

When $x = 1$, the series is $\dfrac{1}{3} - \dfrac{1}{5} + \dfrac{1}{7} - \dfrac{1}{9} + \cdots$

This series converges by the Alternating Series Test.

Therefore, the interval of convergence is $-1 \le x \le 1$.

5 :
- 1 : sets up ratio
- 1 : computes limit of ratio
- 1 : identifies interior of interval of convergence
- 1 : considers both endpoints
- 1 : analysis and interval of convergence

2012 Free Response: BC-6

(b) $\left\lvert g\left(\frac{1}{2}\right) - \frac{17}{120} \right\rvert < \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{224} < \frac{1}{200}$	$2 : \begin{cases} 1 : \text{uses the third term as an error bound} \\ 1 : \text{error bound} \end{cases}$
(c) $g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \cdots + (-1)^n \left(\frac{2n+1}{2n+3}\right) x^{2n} + \cdots$	$2 : \begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$

2012 Free Response: BC-6

Results

Exam	Mean	St Dev	% 9s	% 0s
BC	4.23	2.70	5.3	11.5

- Many students successful in part (a): 4/5 points.
- Difficulty in completing the analysis: application of the Alternating Series Test.
- Part (b): Knew to use the first unused term: third term.
- Part (c): good work in differentiating the series for g.

2012 Free Response: BC-6

Common Errors

- Part (a): algebraic errors in finding a_{n+1}.
  Solving $|x^2| < 1$
  Errors in applying and finding the limit.
  Analysis at the endpoints
  Many did not cite the AST.
  No justification or incorrect reasons involving the harmonic series.

2012 Free Response: BC-6

Common Errors

- Part (b): Considered this a Lagrange error problem.
  Errors in simplification.
- Part (c): Incorrect differentiation of the terms of g and/or the general term.
  Linking unequal terms.
  Summation expression included in the sum.

2012 Free Response: BC-6

To Help Students Improve Performance

- Understand the role of each step in the ratio test.
- Proper notation.
- Solving inequalities, especially those involving a quadratic expression.

AP Calculus Reading

Other Information

- **AP Teacher Community**:
  A new online collaboration space and professional learning network for AP Educators.
  Discussion boards, resource library,
  member directory, email digests, notifications, etc.
  Each community is moderated.
  AP TC by Fall 2012.
  EDG live for 30 days post launch.
- **2013 Reading**:
  June 11-17, Kansas City, Session II

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