Results from the 2011 AP Calculus AB and BC Exams

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APDC

AP Calculus Development Committee

- Co-Chair: Thomas Becvar, St. Louis University High School, St. Louis, MO
- Co-Chair: Tara Smith, University of Cincinnati, Cincinnati, OH
- Robert Arrigo, Scarsdale High School, Scarsdale, NY
- Vicki Carter, West Florence High School, Florence, SC
- Donald King, Northeastern University, Boston, MA
- James Sellers, Penn State University, State College, PA
- Chief Reader: Stephen Kokoska, Bloomsburg University, PA
- College Board Advisor: Kathleen Goto, Iolani School, Honolulu, HI
- ETS Consultants: Fred Kluempen, Craig Wright

APDC

Committee Responsibilities

- Plan, develop, and approve each exam.
- Participate in the Reading.
- Consider statistical analysis associated with the exam.
- Review, revise, and update the Course Description
- Consider the following items.
  1. Special Focus topics.
  2. Advise the Chief Reader on reading issues.
  3. Participate in Outreach Events.
- Four meetings each year. Additional virtual meetings.

Exams

AP Calculus Exams

- Operational: North, Central, and South America (includes Alaska and Hawaii)
- Form A: Alternate Exam, late test
- Form I: International, operational
- Form J: International, alternate
- Section I: Multiple Choice. Section II: Free Response
- Calculator and Non-calculator Sections
- AB and BC Exams
- Common Problems

The Reading

Leadership

- Chief Reader:
  Michael Boardman, Pacific University, OR
- Chief Reader Designate:
  Steve Kokoska, Bloomsburg University, PA
- Assistant to the Chief Reader:
  Tom Becvar, Saint Louis University, MO
- Chief Aide:
  Craig Turner, Georgia College and State University, GA

The Reading

Leadership - Exam Leaders

- AB Exam Leader:
  Jim Hartman, College of Wooster, OH
- BC Exam Leader:
  Peter Atlas, Concord Carlisle Regional High School, MA
- Alternate Exam Leader:
  Stephen Davis, Davidson College, NC
- Overseas Exam Leader:
  Deanna Caveny, College of Charleston, SC

The Reading

Leadership - Question Leaders

- AB1: Particle Motion
  Frances Rouse, UMS-Wright Preparatory School, AL
- AB2/BC2: Model Numerically/Analytically (Tea and Biscuits)
  Ben Klein, Davidson College, NC
- AB3: Tangent Line, Area, Volume
  Pario-Lee Law, D'Evelyn Junior Senior High School, CO
  Dale Hathaway, Olivet Nazarene University, IL
- AB4/BC4: Analysis of a Graph, FTC
  Tara Smith, University of Cincinnati, OH (NSF)
- AB5/BC5: Landfill, Differential Equation
  Virge Cornelius, Lafayette High School, MS
- AB6: Piecewise Defined Function
  Mike Koehler, Blue Valley North High School, KS

The Reading

Leadership - Question Leaders

- BC1: Parametric Particle Motion
  John Jensen, Rio Salado College, AZ
- BC3: Perimeter, Volume
  Tracy Bibelnieks, Augsburg College, MN
- BC6: Taylor Series
  Mark Kannowski, DePauw University, IN
- Alternate Exam TQL:
  Julie Clark, Hollins University, VA
- Overseas Exam TQL:
  Chris Lane, Pacific University, OR

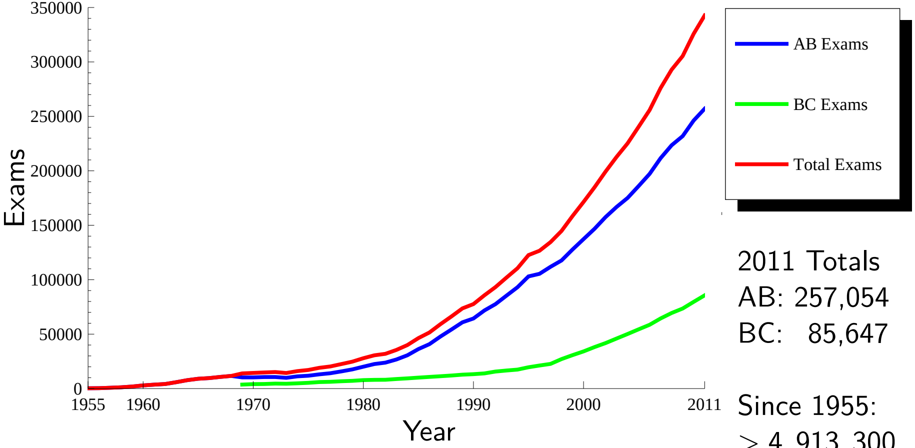
The Reading

Logistics and Participants

- Kansas City:
  Convention Center (versus college campus)
  Westin Hotel (versus college dorms)
- Total Participants: 857
- High School: 49%   College: 51%
- 50 states, DC, and other countries

The Reading

Number of AP Calculus Exams



2011 Totals
AB: 257,054
BC: 85,647

Since 1955:
$> 4,913,300$

The Reading

2011 Scores (Operational Exam)

Operational		
Score	AB	BC
5	21.0%	47.0%
4	16.3%	16.3%
3	18.5%	17.2%
2	10.8%	5.9%
1	33.4%	13.6%
Total	257,054	85,647

The Reading

General Comments

- Students must show their work (bald answers).
- Students must communicate effectively, explain their reasoning, and present results in clear, concise, proper notation.
- Practice in justifying conclusions using calculus arguments.
- More practice with functions represented in graphical and tabular form. Consider piecewise-defined functions.
- Difficulty with application problems, or problems in context.
- Correct units.
- Don't give up on a free response problem. Try all parts.

The Reading

Some Calculator Tips

- Use radian mode.
- Report final answers accurate to three digits to the right of the decimal. Do not round or truncate intermediate values.
- Show the mathematical setup, then use the calculator. Examples:
  - Definite integral: $\int_0^6 |v(t)|\, dt = 12.573$
  - Derivative at a point: $\left.\frac{dy}{dx}\right]_{x=1.2} = 7.658$
- Do not use calculator syntax.
  `fnint(x,x,0,5)`
- Which calculator?

2011 Free Response

General Information

- Six questions on each exam (AB, BC).
- Three common questions: AB2/BC2, AB4/BC4, AB5/BC5.
- Scoring: 9 points for each question.
- Complete and correct answers earn all 9 points.
- The scoring standard is used to assign partial credit.

2011 Free Response

AB1: Particle Motion

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

2011 Free Response

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, since velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t)\rvert\,dt = 12.573$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2011 Free Response

AB1: Particle Motion

Exam	Mean	St Dev	% 9s
AB	2.79	2.44	2.5

- Overall, student performance was poor.
- In general, students did not do well on part (a). Many did not earn either point.
- Most students correctly set up and evaluated the average value integral to earn both points in part (b).
- In part (c), most students failed to earn either point. However, those who successfully set up the integral for total distance usually earned the answer point also.
- In part (d), few students earned all three points.

AB1: Particle Motion

Commons errors:

- Problems entering the correct functions for velocity and acceleration into the calculator, leaving off the $+1$ or misplacing the parentheses on the sine function.
- Considered the sign of one function, velocity or acceleration.
- Use of average acceleration, $\dfrac{v(6) - v(0)}{6 - 0}$ rather than average velocity.
- Ignored the change in direction: $\int_0^6 v(t)\,dt$ instead of $\int_0^6 |v(t)|\,dt$
- Did not use the time $t = 5.196$ to find position.
- Did not use the initial condition $x(0) = 2$.

2011 Free Response

AB1: Particle Motion

To help students improve performance:

- Understand the difference between:
  - Speed and velocity.
  - Average velocity and average acceleration.
  - Total distance traveled and displacement.
- Careful definitions of calculator functions.
- Finding the position of a particle using the Fundamental Theorem of Calculus.

$$x(b) = x(a) + \int_a^b v(t)\,dt$$

AB2/BC2: Model Numerically/Analytically

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

2011 Free Response

(a) $H'(3.5) \approx \dfrac{H(5)-H(2)}{5-2} = \dfrac{52-60}{3} = -2.666$ or -2.667 degrees Celsius per minute	1 : answer
(b) $\dfrac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes. $\dfrac{1}{10}\int_0^{10} H(t)\,dt \approx \dfrac{1}{10}\left(2\cdot\dfrac{66+60}{2} + 3\cdot\dfrac{60+52}{2} + 4\cdot\dfrac{52+44}{2} + 1\cdot\dfrac{44+43}{2}\right) = 52.95$	3 : { 1 : meaning of expression 1 : trapezoidal sum 1 : estimate
(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$ The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.	2 : { 1 : value of integral 1 : meaning of expression
(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$ The biscuits are 8.817 degrees Celsius cooler than the tea.	3 : { 1 : integrand 1 : uses $B(0) = 100$ 1 : answer

AB2/BC2: Model Numerically/Analytically

Exam	Mean	St Dev	% 9s
AB	3.31	2.87	5.5
BC	5.48	2.84	18.0

- Overall performance was average.
- Part (a): most students earned the point.
- Parts (b) and (c): many students had difficulty explaining the meaning of the expressions.
- Part (b): sufficient presentation to convey the trapezoidal rule.
- Part (d): use of a definite integral.

AB2/BC2: Model Numerically/Analytically

Common errors:

- Explanation of expressions involving definite integrals.
- Failure to recognize the time intervals were of unequal lengths.
- Application of the Fundamental Theorem of Calculus.

$$\int_0^{10} H'(t)\,dt = H(10) - H(0)$$

- Part (d): found the temperature of the biscuits, $B(10)$. Did not give a final answer (difference).

AB2/BC2: Model Numerically/Analytically

To help students improve performance:

- The Fundamental Theorem of Calculus in various forms.

  $$f(b) = f(a) + \int_a^b f'(t)\,dt$$

- Application of computational skills to contextual problems.
- Appropriate use of units.

2011 Free Response

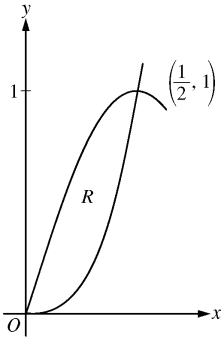
AB3: Tangent Line/Area/Volume

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



2011 Free Response

(a) $f\left(\frac{1}{2}\right) = 1$ $f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$ An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.	$2: \begin{cases} 1: f'\left(\frac{1}{2}\right) \\ 1: \text{answer} \end{cases}$
(b) Area $= \int_0^{1/2} (g(x) - f(x))\,dx$ $= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$ $= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$ $= -\frac{1}{8} + \frac{1}{\pi}$	$4: \begin{cases} 1: \text{integrand} \\ 2: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(c) $\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx$ $= \pi\int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$	$3: \begin{cases} 1: \text{limits and constant} \\ 2: \text{integrand} \end{cases}$

2011 Free Response

AB3: Tangent Line/Area/Volume

Exam	Mean	St Dev	% 9s
AB	4.64	2.69	7.7

- Overall student performance was good, especially in parts (a) and (b).
- Part (a): most students earned both points.
- Part (b): most students earned the setup point and at least one antiderivative point.
- Part (c): the most difficult for students.

2011 Free Response

AB3: Tangent Line/Area/Volume

Commons errors:

- Antiderivative of $\sin(\pi x)$
- Reversal: $\int_0^{1/2} (f(x) - g(x))\,dx$

  Negative answer to positive area.
- Part (c): incorrect placement of parentheses and square of the wrong term(s).

2011 Free Response

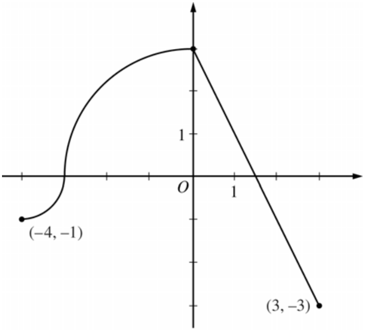
AB3: Tangent Line/Area/Volume

To help students improve performance:

- In addition to formulas for the volume of a solid of revolution, consider the volume of a solid with known cross-sectional area.
- Mathematical notation: missing, misplaced symbols, poor presentation.
- Present intermediate work.

AB4/BC4: Graphical Analysis

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

2011 Free Response

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore, g has an absolute maximum at $x = \frac{5}{2}$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus, the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is

$\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

2011 Free Response

AB4/BC4: Graphical Analysis

Exam	Mean	St Dev	% 9s
AB	2.44	2.26	0.4
BC	4.13	2.29	1.6

- Student performance was below expectation.
- Part (a): many students earned only one of three points.
- Part (b): Lack of justification for an absolute maximum.
- Part (c): few students earned the point.
- Part (d): many correctly found the average rate of change of f. Few could explain why the statement does not contradict the Mean Value Theorem.

2011 Free Response

AB4/BC4: Graphical Analysis

Commons errors:

- Part (a):
  - $\int_0^{-3} f(t)\,dt = -\int_{-3}^{0} f(t)\,dt$
  - Ignored $2x$ when finding $g'(x)$.
  - Used 3 instead of -3.
- Part (b): Justification for a local maximum rather than a global maximum.
- Part (c):
  - Listed $x = -3$ as an inflection point.
  - Rejected $x = 0$ (because $g''(x) = f'(x)$ DNE).
- Part (d): application of the Mean Value Theorem.

2011 Free Response

AB4/BC4: Graphical Analysis

To help students improve performance:

- Understand the difference between a local extrema and a global extrema. And, be able to use a variety of methods to justify.
- Communicate mathematics using clear, mathematically correct, precise language.
- Practice with functions whose definitions include an integral.
- Be able to reason from the graph of a function, the graph of a derivative, or a function whose definition involves a presented graph.

AB5/BC5: Modeling, Differential Equation

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

2011 Free Response

(a) $\left.\frac{dW}{dt}\right|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$

The tangent line is $y = 1400 + 44t$.

$W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons

$2 : \begin{cases} 1 : \frac{dW}{dt} \text{ at } t = 0 \\ 1 : \text{answer} \end{cases}$

(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$

Therefore, $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$.

The answer in part (a) is an underestimate.

$2 : \begin{cases} 1 : \frac{d^2W}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$

(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$

$\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$

$\ln|W - 300| = \frac{1}{25}t + C$

$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$

$W - 300 = 1100e^{\frac{1}{25}t}$

$W(t) = 300 + 1100e^{\frac{1}{25}t}, \quad 0 \leq t \leq 20$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } W \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

2011 Free Response

AB5/BC5: Modeling, Differential Equation

Exam	Mean	St Dev	% 9s
AB	1.63	2.32	0.5
BC	3.53	2.91	2.4

- Overall student performance was very poor.
- For those who earned points, generally both in part (a) - tangent line approximation.
- Most students failed to earn any points in part (b).
- Part (c) was a traditional separable DE. Most common - all 5 or middle 3.

2011 Free Response

AB5/BC5: Modeling, Differential Equation

Commons errors:

- Algebra and arithmetic errors throughout the problem.
- Part (a): string of equalities.
  Example: $(1/25)(1100) = 44(1/4) = 11 + 1400 = 1411$
- Part (b): $\dfrac{d^2W}{dt^2} = \dfrac{1}{25}$ (incorrect)
  $\dfrac{d^2W}{dt^2} > 0$ at a single point rather than on the entire interval.
- Part (c): errors in separating variables.

2011 Free Response

AB5/BC5: Modeling, Differential Equation

To help students improve performance:

- Solving DEs in contextual problems.
- The behavior of a solution to a DE without actually solving the DE.
- Solving DEs in which the independent variable does not appear.
- Determination of an underestimate or an overestimate requires knowledge of the behavior of the function and its derivatives on an entire interval, not at a single point.

AB6: Analysis of a Piecewise Defined Function

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

(a) $\lim\limits_{x\to 0^-} (1 - 2\sin x) = 1$ $\lim\limits_{x\to 0^+} e^{-4x} = 1$ $f(0) = 1$ So, $\lim\limits_{x\to 0} f(x) = f(0)$. Therefore, f is continuous at $x = 0$.	2 : analysis

2011 Free Response

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$-2\cos x \neq -3$ for all values of $x < 0$.

$-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$.

Therefore, $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

(c)
$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$
$$= \int_{-1}^{0} (1 - 2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx$$
$$= \Big[x + 2\cos x\Big]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x}\right]_{x=0}^{x=1}$$
$$= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)$$

$$\text{Average value } = \frac{1}{2}\int_{-1}^{1} f(x)\,dx$$
$$= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$$

$4 : \begin{cases} 1 : \int_{-1}^{0} (1 - 2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

AB6: Analysis of a Piecewise Defined Function

Exam	Mean	St Dev	% 9s
AB	3.03	2.32	0.5

- Students did reasonably well, given a piecewise defined function.
- Part (a): most students earned one of the two points.
- Part (b): many students differentiated the expressions but did not express $f'(x)$ as a piecewise defined function. Few students earned the third point.
- Part (c): many students earned the first point and one of the two antiderivative points.

2011 Free Response

AB6: Analysis of a Piecewise Defined Function

Commons errors:

- Part (a): attempt to show continuity by evaluating the function at each piece (1/2). Need to use limits to demonstrate continuity.
- Part (b): Failure to express $f'(x)$ as a piecewise defined function. Attempt to solve $-2\cos x = -3$ and presentation of an answer.
- Part (c): average rate of change of f: $\dfrac{f(1) - f(-1)}{2}$ rather than the average value of f.
  Few students earned the fourth point.

2011 Free Response

AB6: Analysis of a Piecewise Defined Function

To help students improve performance:

- Correctly use the definition of continuity.
  $\lim_{x \to a} f(x) = f(a)$
- Piecewise defined functions.
- Computation of derivatives and antiderivatives analytically.

2011 Free Response

BC1: Parametric Particle Motion

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin\left(t^2\right)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007 Acceleration $= \langle x''(3), y''(3)\rangle$ $= \langle 4, -5.466\rangle$ or $\langle 4, -5.467\rangle$	$2: \begin{cases} 1: \text{speed} \\ 1: \text{acceleration} \end{cases}$
(b) Slope $= \dfrac{y'(3)}{x'(3)} = 0.031$ or 0.032	1 : answer
(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt}\, dt = 21$ $y(3) = -4 + \int_0^3 \frac{dy}{dt}\, dt = -3.226$ At time $t = 3$, the particle is at position $(21, -3.226)$.	$4: \begin{cases} 2: x\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \\ 2: y\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \end{cases}$
(d) Distance $= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 21.091$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

2011 Free Response

BC1: Parametric Particle Motion

Exam	Mean	St Dev	% 9s
BC	5.24	2.64	13.9

- Student performance was very good.
- Most students earned all points in parts (a) and (b).
- Part (c): many students earned the two points for the x-coordinate, but did not earn points for the y-coordinate.
- Part (d): if correct setup, then correct answer.

BC1: Parametric Particle Motion

Commons errors:

- Part (a): $\text{Speed} = \dfrac{y'(3)}{x'(3)}$
- Part (b): speed as the slope of the tangent line.
- Part (c): many students tried to find an atiderivative for $\sin t^2$ rather than use a calculator to evaluate the definite integral.
  Failure to use $y(0) = -4$.

BC1: Parametric Particle Motion

To help students improve performance:

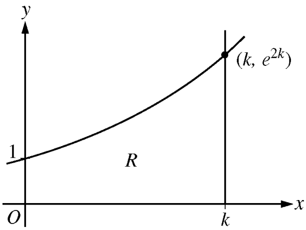
- Practice with particle motion, including computations of speed, velocity, and distance traveled.
- The Fundamental Theorem of Calculus written as:

  $$f(b) = f(a) + \int_a^b f'(t)\,dt$$

- Show intermediate work.
- Careful attention to writing complex expressions (including parentheses).

BC3: Perimeter, Volume

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.



(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

(c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

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(a) $f'(x) = 2e^{2x}$ Perimeter $= 1 + k + e^{2k} + \int_0^k \sqrt{1 + \left(2e^{2x}\right)^2}\, dx$	$3: \begin{cases} 1: f'(x) \\ 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^k \left(e^{2x}\right)^2 dx = \pi \int_0^k e^{4x}\, dx = \frac{\pi}{4} e^{4x} \Big\vert_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$	$4: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(c) $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$ When $k = \frac{1}{2}$, $\frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$.	$2: \begin{cases} 1: \text{applies chain rule} \\ 1: \text{answer} \end{cases}$

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BC3: Perimeter, Volume

Exam	Mean	St Dev	% 9s
AB Part	3.84	1.91	
BC Part	1.67	1.27	
Total	5.51	2.74	19.5

- Students performed well, mode score 9.
- Part (a): almost half of all students earned all three points.
- Part (b): most students earned the first two points. The antiderivative and answer were more difficult.
- Part (c): errors in the computation of the derivative. Could use integral setup or answer in part (b).

2011 Free Response

BC3: Perimeter, Volume

Commons errors:

- Part (a): incorrect integral for arc length, no integral for arc length, or leaving out other boundaries for R.
- Part (b): integral expression, but no evaluation.
  Incorrect antiderivative.
- Part (c): incorrect application of the chain rule.
  Derivative of e^{4x}.

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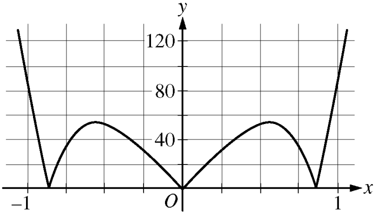
BC3: Perimeter, Volume

To help students improve performance:

- Differentiation and integration of functions of the form e^{ax}.
- Intermediate steps and communication of the final answer.
- Inclusion of the terminating differential (dx).
  Separates the integral from the remaining expression.
  Indicates the variable of integration.

BC6: Taylor Series



Graph of $y = \left| f^{(5)}(x) \right|$

Let $f(x) = \sin\left(x^2\right) + \cos x$. The graph of $y = \left| f^{(5)}(x) \right|$ is shown above.

(a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$, and write the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x = 0$.

(b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$. Use this series and the series for $\sin\left(x^2\right)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x = 0$.

(c) Find the value of $f^{(6)}(0)$.

(d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = \left| f^{(5)}(x) \right|$ shown above, show that $\left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}$.

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(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\sin\left(x^2\right) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots$

$3 : \begin{cases} 1 : \text{series for } \sin x \\ 2 : \text{series for } \sin\left(x^2\right) \end{cases}$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$f(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{121x^6}{6!} + \cdots$

$3 : \begin{cases} 1 : \text{series for } \cos x \\ 2 : \text{series for } f(x) \end{cases}$

(c) $\frac{f^{(6)}(0)}{6!}$ is the coefficent of x^6 in the Taylor series for f about $x = 0$.

Therefore, $f^{(6)}(0) = -121$.

1 : answer

(d) The graph of $y = \left|f^{(5)}(x)\right|$ indicates that $\max\limits_{0 \le x \le \frac{1}{4}} \left|f^{(5)}(x)\right| < 40$.

Therefore,

$$\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \le \frac{\max\limits_{0 \le x \le \frac{1}{4}} \left|f^{(5)}(x)\right|}{5!} \cdot \left(\frac{1}{4}\right)^5 < \frac{40}{120 \cdot 4^5} = \frac{1}{3072} < \frac{1}{3000}.$$

$2 : \begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$

2011 Free Response

BC6: Taylor Series

Exam	Mean	St Dev	% 9s
BC	3.97	2.26	1.2

- Better on this series question than in previous years. Mode score 5.
- Part (a): most students earned both points.
- Part (b): most students earned the first point, and at most 1 of the next 2.
- Part (c): most did not earn the point.
- Part (d): 0 or 2. Understood the error bound, or not.

BC6: Taylor Series

Commons errors:

- Series for $\sin x$ and $\cos x$.
- Series manipulation: substituting x^2 for x.
- Part (b): combining of like terms in order to present the first four nonzero terms of the Taylor series for f.
- Part (c): use of the coefficient on x^6 to determine $f^{(6)}(0)$.
- Part (d): many students did not know the Lagrange error bound. Difficulty in finding a correct value for $\max\limits_{0\leq x\leq 1/4}\left|f^{(5)}(x)\right|$

2011 Free Response

BC6: Taylor Series

To help students improve performance:

- Know the power series expansions for common functions, including $\sin x$ and $\cos x$.
- Creating new series from old series using substitution and algebraic manipulation.
- Practice with error bounds.

AP Calculus Reading

Other Information

- AP Central:
  - Course Description and Teacher's Guide
  - Newsletter, Course Perspective, and Audit information
  - Released Exams, Free Response Questions, Special Focus Material, Curriculum Modules, Lesson Plans, Teaching Strategies, and Course Content Related Articles
  - 2008 released exam, for sale
- Free response questions: 2 calculator, 4 non-calculator
- Rights-only scoring on the multiple choice quesitons
- 2012 Reading: June 10-16, Kansas City, Session II