AP Calculus: Facts, Figures, and FAQs

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Objectives

1. The AP Calculus Exam and the Reading.
2. The AP Calculus Exam is graded fairly, accurately, and consistently.

   We award points for good calculus work.
3. Please complete a Reader application.

   A unique professional development opportunity.

   http://apcentral.collegeboard.com

The AP Calculus Exam

AP Calculus Exams

- Operational: North, Central, and South America (includes Alaska and Hawaii)
- Form A: Alternate Exam, late test
- Form I: International, operational
- Form J: International, alternate
- Section I: Multiple Choice. Section II: Free Response
- Calculator and Non-calculator Sections
- AB and BC Exams
- Common Problems
- Multiple choice: rights only scoring

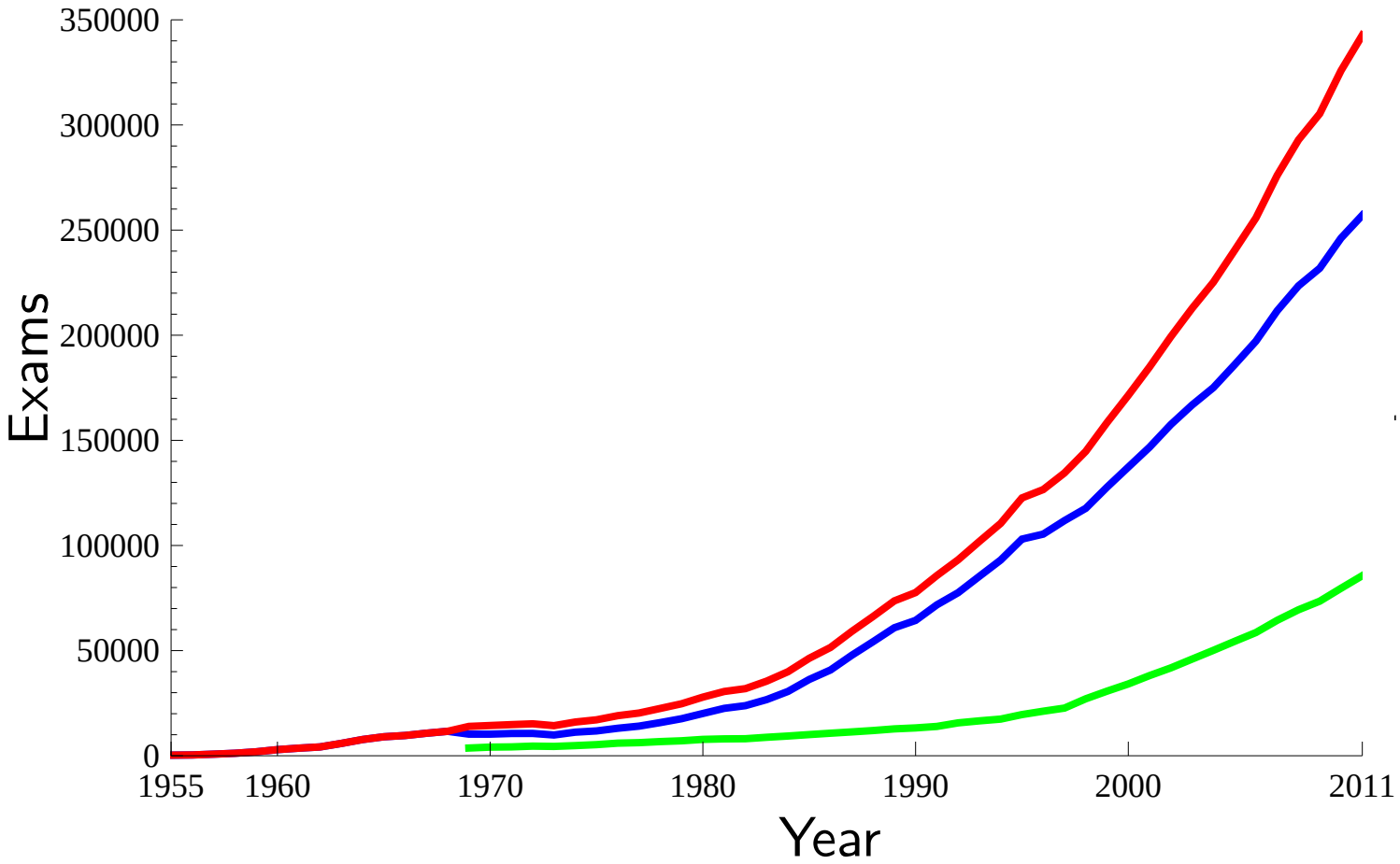
The AP Calculus Exam

Free-Response Questions

- Six questions on each exam (AB and BC)
- Three questions in common to AB and BC
- Two calculator active questions on each exam
- Nine points allotted to each problem
- We award credit for complete and correct work.
- Scoring rubric (standard) is used to assign partial credit.

The AP Calculus Exam

Number of AP Calculus Exams



2011 Totals
AB: 257,054
BC: 85,647

Since 1955:
$> 4,913,300$

The Reading

Background Information

- Score the free-response questions
- Kansas City:
  Convention Center (versus college campus)
  Westin Hotel (versus college dorms)

  Previous locations: Louisville, Colorado State University, Clemson University, Rider University, Trenton State
- Seven day Reading (five day pre-reading preparation)
- Total Participants: 857
- High School: 49% College: 51%
- 50 states, DC, and other countries
- Everyone reads three questions over seven days

The Reading

Positions

- Chief Reader
- Exam Leaders (AB, BC, Alternate, International)
- Question Leaders
- Question Team Members
- Table Leaders
- Readers (18 per room, 2 TLs)

2011 Free Response

AB1: Particle Motion

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

2011 Free Response

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5,$ since velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Distance $= \int_0^6 |v(t)|\,dt = 12.573$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.
$v(t)$ changes sign from positive to negative at time $t = b$.
$x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135

$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2011 Free Response

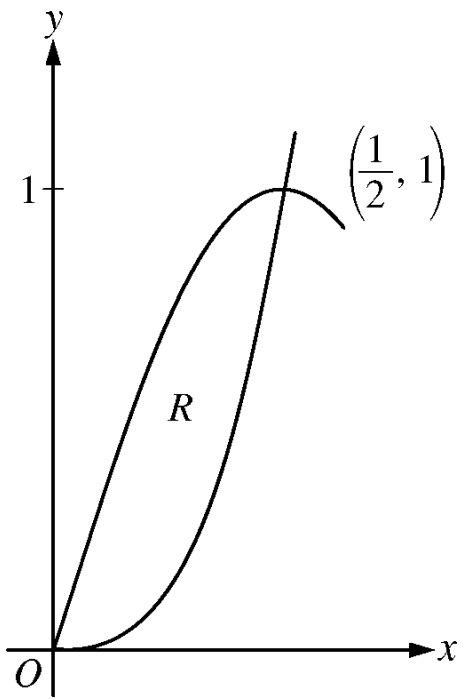
AB3: Tangent Line/Area/Volume

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



2011 Free Response

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b) $\text{Area} = \int_0^{1/2} (g(x) - f(x))\, dx$

$= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$

$= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$

$= -\frac{1}{8} + \frac{1}{\pi}$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\pi \int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx$

$= \pi \int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

2011 Scores (Operational Exam)

Operational		
Score	AB	BC
5: Extremely well qualified	21.0%	47.0%
4: Well qualified	16.3%	16.3%
3: Qualified	18.5%	17.2%
2: Possibly qualified	10.8%	5.9%
1: No recommendation	33.4%	13.6%
Total	257,054	85,647

Notes

- Equate items from previous years
- Statisticians from the College Board and ETS
- Test development specialists from ETS
- College comparability study