

TI-83/84 Graphing Calculator Manual

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Chapter 1

Introduction to the TI-83 Plus

This chapter is a brief introduction to the TI-83 Plus and the use of calculator lists. Some basic commands and techniques are discussed that are used throughout this manual. More detailed descriptions of built-in calculator functions are given in the TI-83 Plus Graphing Calculator Guidebook.

The keypad on the TI-83 Plus is almost identical to the original TI-83. On the TI-83 Plus there is a new, purple **APPS** key, the **MATRIX** menu is a **2nd** (yellow, or secondary) function, and the **FINANCE** package is found in the **APPS** menu. The number keys are white, larger, and easier to use.

The TI-83 Plus is a *Flash* calculator. As new software becomes available, it may be downloaded to the TI-83 Plus (and installed) in order to upgrade the machine. The TI-83 Plus has a more flexible and powerful archive feature. Collections of programs, lists, equations, etc. may be stored as a *group* and easily recalled or transferred to other calculators. In addition, the TI-83 Plus has almost seven times the memory of the TI-83. This allows the user to store more (and larger) data sets, programs, and applications.

On, Off, Contrast, Memory: The **ON** key is, of course, used to turn the calculator on, and the key sequence **2nd**; **OFF** is used to turn the machine off. There is also a battery saving feature built into the TI-83 Plus that will automatically turn the machine off after approximately 5 minutes of inactivity.

The yellow **2nd** key and the blue up and down arrows (in the top right portion of the calculator keypad) are used to adjust the screen contrast.

The keystrokes $\boxed{2\text{nd}};$ $\boxed{\Delta}$ darken the screen while $\boxed{2\text{nd}};$ $\boxed{\nabla}$ makes the screen lighter. To save keystrokes, press $\boxed{2\text{nd}}$ and hold down $\boxed{\Delta}$ (or $\boxed{\nabla}$). This keystroke technique will continuously darken (or lighten) the screen. Release either arrow key when you have reached the desired screen contrast. As the contrast is adjusted there is a single digit, 0–9, that appears in the top right corner of the screen. This number is an indication of the screen contrast (the higher the number, the darker the screen), and can be used to gauge the energy level of the batteries in the calculator. While the TI-83 Plus does display a warning message if the batteries are low, if the contrast is set to 9 and the screen is still very light, this is also an indication that the four AAA batteries need to be replaced.



Figure 1.1: MEMORY menu.

All real and complex variables, lists, matrices, Y-variables, programs, pictures, graph databases, groups, and strings stored in the calculator memory may be (selectively) deleted. In addition, the calculator memory may be checked or reset. The MEMORY menu (obtained by using $\boxed{2\text{nd}};$ $\boxed{\text{MEM}}$) is shown in Figure 1.1. The option `ClrAllLists` is used to simultaneously clear all calculator lists and is particularly useful for beginning a new problem.

Home Screen: If you think of all the menus and functions of the TI-83 Plus as being part of a huge tree diagram, then the Home Screen is the top-most, or main, screen. It is familiar, comfortable ground, and instructions and programs are executed from this screen. The blinking rectangular cursor (on any screen, or branch) indicates the calculator is in *typeover* mode and is ready to accept commands. To return to the Home Screen from any other screen, use $\boxed{2\text{nd}};$ $\boxed{\text{QUIT}}$.

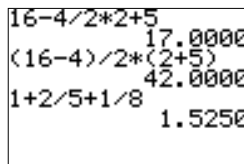


Figure 1.2: Arithmetic calculations.

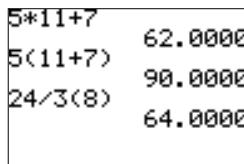


Figure 1.3: Standard versus implied multiplication.

Arithmetic Calculations: The standard arithmetic operations are located on the right column of the calculator keypad: $\boxed{+}$ (addition), $\boxed{-}$ (subtraction), $\boxed{\times}$ (multiplication), $\boxed{\div}$ (division), and $\boxed{\wedge}$ (exponentiation). The usual hierarchy of operations applies and where there is a tie, the TI-83 Plus evaluates expressions left to right. Parentheses should be used freely in arithmetic expressions to ensure the desired order of evaluation, and the $\boxed{(-)}$ key is used for negation. After typing an arithmetic expression (or command), press $\boxed{\text{ENTER}}$ to evaluate. Figure 1.2 illustrates several arithmetic calculations. The evaluation rules for implied multiplication are slightly different from those on the TI-82. Figure 1.3 demonstrates the use of standard versus implied multiplication.

Note! While bad mathematical practice, you may omit the close, or right, parenthesis *at the end* of an arithmetic expression. The TI-83 Plus will automatically close all open parentheses. \diamond

Last Entry, Last Answer: Every entry, or expression, is automatically

stored in the TI-83 Plus memory (subject to space limitations), even as entries scroll off the screen and even after the calculator is turned off. To recall, or redisplay, the most recent, or last, entry on the calculator screen, use $\boxed{2\text{nd}}$; $\boxed{\text{ENTRY}}$. To access earlier entries, repeat the keystrokes $\boxed{2\text{nd}}$; $\boxed{\text{ENTRY}}$ as necessary until the desired entry is displayed. The variable ANS always contains the value of the most recent, or last, calculation. The *last answer* variable is accessed by using $\boxed{2\text{nd}}$; $\boxed{\text{ANS}}$ and this token (ANS) may be part of any arithmetic calculation.

Menus: The menu keys on the calculator keypad are used to access built-in functions, operations, and settings not found on any of the fixed keys. If you press a menu key, one or more menus will be displayed on the top line of the calculator screen. The current, or active, menu will be highlighted, or darkened (Figure 1.4). The blue left and right arrow keys, $\boxed{\leftarrow}$ and $\boxed{\rightarrow}$, move the top line cursor to the other main menus. To select from a menu, press the number of the item desired, or move the cursor up or down with the blue arrow keys ($\boxed{\uparrow}$ and $\boxed{\downarrow}$) to highlight the desired selection, and press $\boxed{\text{ENTER}}$.



Figure 1.4: The $\boxed{\text{STAT}}$; $\boxed{\text{EDIT}}$ menu.

Note! The TI-83 Plus has two special features for quickly viewing and selecting menu choices. These shortcuts are designed to save keystrokes.

- (N1) If the left-most top line menu is highlighted, pressing the blue left arrow, $\boxed{\leftarrow}$, will cause the cursor to *wrap around* and highlight the right-most top line menu. Similarly, you can *wrap around* from the right to the left by using $\boxed{\rightarrow}$ from the right-most top line menu.
- (N2) If a top line menu has more than a screen-full of options (indicated by a down arrow (\downarrow) on the bottom line item, press $\boxed{\text{ALPHA}}$; $\boxed{\downarrow}$ to move down one screen at a time. Similarly, pressing $\boxed{\text{ALPHA}}$; $\boxed{\uparrow}$ will move the cursor up one screen. You can also wrap around the top (bottom) of a menu list by using the up (down) arrow from the first (last) entry in a menu list. \diamond

Display Modes: As the name suggests, the $\boxed{\text{MODE}}$ key is used to view and/or change certain calculator *modes*, for example, settings that determine the way numbers and graphs are displayed on the screen. It is also used to control other calculator configuration options, for example *radian* versus *degree* mode (Figure 1.5). The settings in effect are highlighted, or darkened. To change/select a setting, move the blinking cursor with the blue arrow keys to the desired option and press $\boxed{\text{ENTER}}$ to highlight that option.

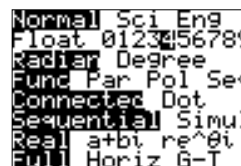


Figure 1.5: $\boxed{\text{MODE}}$ settings.

Functions: To define a function, or Y-VARS , press the $\boxed{\text{Y=}}$ key. Using the Y= editor you can store up to 10 functions. A function may be defined in terms of X (function graphing), T (parametric graphing), θ (polar graph-

```

Plot1 Plot2 Plot3
\Y1 X^2+4X+4
\Y2 1/√(2π)*e^(-
X^2)
\Y3 sin(X)^2-cos(
X)^2
\Y4 =
\Y5 =

```

Figure 1.6: The **Y=** editor.

```

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=0
Ymax=100
Yscl=10
Xres=1

```

Figure 1.7: **WINDOW**; WINDOW settings.

```

EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg
A:PwrReg
B:Logistic
C:SinReg

```

Figure 1.8: **STAT**; CALC menu.

```

EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
8:TInterval...
9:2-SampZInt...
0:2-SampTInt...
A:1-PropZInt...
B:2-PropZInt...
C:χ²-Test...
D:2-SampFTest...
E:LinRegTTest...
C:ANOVA(

```

Figure 1.9: **STAT**; TESTS menu.

ing), or $U(n)$ and $V(n)$ (sequence graphing). The independent variable, or type of function, is determined by the mode settings. Figure 1.6 shows several functions defined in terms of X . The icon to the left of each variable in the **Y=** editor indicates the graph style (line, dot, thickness, etc.). To change a graph style, position the cursor on the icon and press **ENTER** to toggle through the options. Statistical plots may also be turned on or off from within the **Y=** editor.

Plotting Graphs: The **GRAPH** key (in the upper right part of the calculator keypad) is used to display the graph of a function, a sequence, or a statistical plot. The plot region, or viewing window, is set in the **WINDOW**; **WINDOW** menu (Figure 1.7). The tokens **Xmin** and **Xmax** set the minimum and maximum value on the x (horizontal) axis. **Xscl** is the distance between tick marks on the x axis. **Ymin**, **Ymax**, and **Yscl** work similarly for the y axis. The token **Xres** controls the pixel resolution for function graphs only. If **Xres=4**, for example, functions are evaluated at every 4th horizontal pixel. Smaller values for **Xres** produce smoother, but slower, calculator graphs. The value must be an integer from 1 to 8; the default value is 1. The seven tokens, **Xmin**, **Xmax**, **Xscl**, **Ymin**, **Ymax**, **Yscl**, and **Xres**, are collectively referred to as the window settings.

The **ZOOM**; **ZOOM** menu contains options for *automatic* window settings. These include a standard viewing window $([-10, 10] \times [-10, 10])$, a *friendly window* $([-4.7, 4.7] \times [-3.1, 3.1])$, zooming in/out, and options for graphs of trigonometric functions and for statistical plots. The **TRACE** key is used to *move around on* graphs displayed on the screen. If you press this key while a graph is on the screen, the calculator displays a blinking cross on a graph and *coordinates* of this cross at the bottom of the screen. The blue left and right arrows may be used to move the blinking cross along the graph, and the up and down arrows may be used to move among graphs.

Built-in Statistics: The TI-83 Plus has several built-in functions for analyzing data. Many are contained in the **STAT**; **CALC** (Figure 1.8) and the **STAT**; **TESTS** (Figure 1.9) menus. Some of the functions are: summary statistics for a single data set or two sets simultaneously, various regression *lines*, large and small sample confidence intervals for a population mean, various hypothesis tests, and analysis of variance.

Other built-in statistical functions are in the menus **DISTR**; **DISTR** (Figure 1.10) and **DISTR**; **DRAW** (Figure 1.11). These menus include choices for computing values of common probability distribution functions (pdf) and cumulative distribution functions, and for illustrating probability by shading area under a pdf.

Plotting Statistical Data: The **STATPLOT**; **STATPLOTS** menu is used

to access statistical plot options and to turn on/off all statistical plots (Figure 1.12). One, two, or three statistical plots may be displayed on the screen simultaneously. The TI-83 Plus can display scatterplots, line graphs, two types of boxplots, histograms, and normal probability plots.

Programs: The `PRGM`; EXEC, EDIT, and NEW menus are used to execute existing calculator programs stored in memory, edit existing programs, and to create new programs, respectively. In this manual, all programs will be executed from the Home Screen. From the Home Screen, press `PRGM` and in the EXEC menu select the desired program (by highlighting or pressing the number or letter that corresponds to the desired program). (Programs are sorted alphanumerically.) This keystroke sequence causes the TI-83 Plus to paste a copy of the program name on the Home Screen (together with the calculator function `prgm`). To execute, or run, the program, press `ENTER`.

Note! The *screen up/down* and *wrap around* features work in the `PRGM`; EXEC and `PRGM`; EDIT menus. These shortcuts are very useful when searching a long list of programs. ◇

Lists: A TI-83 Plus list is a set, or collection, of numbers, or observations. A list can hold up to 999 numerical values and is the principal way to store data (for analysis) on the TI-83 Plus. Many of the built-in statistical functions and programs *operate* on or use data stored in a list (or lists) during execution.

There are six simple built-in names for lists: L1, L2, L3, L4, L5, and L6. These list names, or tokens, are accessed using the keystrokes `2nd`; L1, or `2nd`; L2, etc. You may also create and use lists with a descriptive name for easy identification. The name must be a string of up to 5 characters. The first character must be a letter which may be followed by letters and numbers, including the greek letter θ . The number of lists is limited by the available memory. There are two primary *places* in the TI-83 Plus for creating and editing a list: on the Home Screen, or in the stat list editor (accessed using `STAT`; EDIT; EDIT).

Example: Coffee has become a staple at the breakfast table and throughout the day for many Americans. In a recent survey, ten men and seven women were asked, “How many cups of coffee do you drink per day?” The results are given in the table below.

Group	Number of Cups of Coffee									
Men	5	2	1	7	7	4	3	5	6	2
Women	2	2	3	1	1	1	1	8		

Create list L1 on the Home Screen so that it contains the data from



Figure 1.10: `DISTR`; DISTR menu.



Figure 1.11: `DISTR`; DRAW menu.



Figure 1.12: `STATPLOT`; STATPLOTS menu.

Men. Also on the Home Screen, create a list named WOMEN containing the numerical values for women.

Figure 1.13: Creating and displaying list L1 on the Home Screen.

Figure 1.14: Use of list subscript notation.

Figure 1.15: Creating a named list.

Figure 1.16: Clearing lists.

Figure 1.17: Accessing the stat list editor.

Calculator Solution:

(S1) On the Home Screen, curly braces ($\{ \}$) are used to denote, or enclose, lists. Numbers in a list created on the Home Screen are separated by commas. Type the numerical values for Men within curly braces separated by commas. Finish the expression by using the keystrokes $\boxed{\text{STO}\rightarrow}$; $\boxed{\text{L1}}$; $\boxed{\text{ENTER}}$ to store the data in list L1 (Figure 1.13). The list may be displayed on the Home Screen by pressing $\boxed{\text{L1}}$; $\boxed{\text{ENTER}}$. Spaces rather than commas separate values in a *displayed* list.

Note!

(N1) After you press $\boxed{\text{ENTER}}$ the TI-83 displays the contents of the list.

(N2) The three dots at the end of the displayed list indicate there is more data in the list that cannot be included on one calculator screen line. To view other entries in the list, *scroll* through the data by using the blue left and right arrow keys ($\boxed{\leftarrow}$ and $\boxed{\rightarrow}$).

(N3) You can edit a single entry, or row, in a list by using *subscript* notation, for example, L1(5) denotes the 5th row in list L1 (Figure 1.14). \diamond

(S2) Type the values for Women within curly braces separated by commas. Complete the expression by using the $\boxed{\text{STO}\rightarrow}$ key, followed by the name of the list, and $\boxed{\text{ENTER}}$ (Figure 1.15).

Note!

Named lists used in any *calculation* must include the small L in front of the name to indicate a list. Otherwise the TI-83 Plus interprets the list name as (the product of) single variables in an arithmetic expression. The symbol L is included in front of the name when choosing from the $\boxed{\text{LIST}}$; NAMES menu, or just the symbol L can be copied from the $\boxed{\text{LIST}}$; OPS menu. \diamond \square

To clear all of the entries in a list, or lists, from the Home Screen use $\boxed{\text{STAT}}$; $\boxed{\text{EDIT}}$; $\boxed{\text{CLRList}}$ (Figure 1.16). If you clear the rows of a named list, the TI-83 does not delete the list name. A named list can be deleted using $\boxed{\text{MEM}}$; Mem Mgmt/Del; List.

The stat list editor is another way to create and edit TI-83 Plus lists. To access this feature use $\boxed{\text{STAT}}$; $\boxed{\text{EDIT}}$; $\boxed{\text{EDIT}}$ (Figure 1.17). Lists may be included as part of mathematical expressions and functions, and may be referred to by programs.

Example: The table below lists the mean costs for textbooks per year reported by students majoring in different academic disciplines.

Major	Average Cost for Textbooks
Accounting	\$221.80
Biology	250.95
Psychology	175.66
Mathematics	275.60
Economics	234.50
Music	191.75
Philosophy	288.55

Use the stat list editor to store this data into list L1. Divide each entry in this list by 2 (to obtain an estimate for the mean cost per semester) and place the new list in L2.

Calculator Solution:

- (S1) Clear the entries in list L1 and access the stat list editor.
- (S2) Use the blue arrow keys to place the solid rectangular cursor on the first row of list L1. This cursor indicates the *active* list row. As you type an entry it will appear on the bottom line of the stat list editor. Press **ENTER** to place the entry into the list and to make the next row active (Figure 1.18).

Note!

- (N1) Do not use commas in numbers on the TI-83. This will produce a **SYNTAX** error message.
- (N2) Data must be added in ascending row order in a list. In the stat list editor, you cannot position the cursor beyond the first empty row in any list.
- (N3) To edit existing data in a list, position the cursor on the desired row. Type the correct number and press **ENTER**.
- (N4) To delete data in a list, position the cursor on the desired row and press **DEL**. This keystroke sequence deletes the row and *closes up* the list.
- (N5) To place additional data in a list (other than at the bottom), position the cursor on the row following the number to be inserted. Press **INS** to *open up* the list. A 0 is initially placed in the new, inserted row. Type the desired, additional number and press **ENTER** to insert it into the list. ◇

- (S3) Divide each entry in list L1 by 2, and store the resulting list in L2 (Figure 1.19). The resulting list is displayed on the Home Screen

L1	L2	L3	1
250.95	-----	-----	
175.66			
275.60			
234.50			
191.75			
288.55			

L1()=221.8

Figure 1.18: List L1 in the stat list editor.

(L1/2)→L2
(110.95 125.48 ...)

Figure 1.19: Use of lists in an arithmetic expression.

L1	L2	L3	1
250.95	110.90	-----	
175.66	125.48		
275.60	87.83		
234.50	137.80		
191.75	95.88		
288.55	144.28		
LN(1)=221.8			

Figure 1.20: Resulting list viewed in the stat list editor.

L5	L6	NAME	0
-----	-----		
Name=			

Figure 1.21: Creating a named list in the stat list editor.

L5	L6	INCH	?
-----	-----		
INCH=12*LFEET			

Figure 1.22: A list as the result of an arithmetic calculation, in the stat list editor.

SetupEditor	LCLE
RK, LJAIL, L1	Done

Figure 1.23: Use of SetupEditor to control the lists in the stat list editor.

CLERK	JAIL	L1	1
4.00	21.00	221.80	
6.00	20.00	250.95	
6.00	20.00	175.66	
2.00	19.00	275.60	
15.00	18.00	234.50	
4.00	23.00	191.75	
4.00	20.00	288.55	
CLERK(1)=4			

Figure 1.24: The resulting stat list editor.

and may also be viewed in the stat list editor (Figure 1.20).

Note! Named lists may also be created, viewed, and edited in the stat list editor.

- (N1) To create a named list, place the cursor on the top line (the line containing the names) and use the right arrow key (\rightarrow) to move to an unnamed column. Name= will appear on the bottom line of the stat list editor and the alpha (green) characters become active (as indicated by the alpha cursor in the top right portion of the calculator screen). You can then type the desired name of a list, followed by $\boxed{\text{ENTER}}$ (Figure 1.21).
- (N2) To make a named (or built-in) list the result of an arithmetic expression, position the cursor on the name. Type the arithmetic expression and press $\boxed{\text{ENTER}}$ (Figure 1.22).
- (N3) If a named list is created as a result of an arithmetic calculation, its contents are not immediately visible in the stat list editor. To view a named list created in this way, position the cursor on the top line of an unnamed list. Type the name of the list followed by $\boxed{\text{ENTER}}$. The numbers stored in the named list will then be displayed.
- (N4) The command SetupEditor (accessed through $\boxed{\text{STAT}}$; EDIT) is used to control the lists displayed in the stat list editor. SetupEditor followed by a sequence of (up to 20) lists replaces the stat list editor with the new sequence of lists (Figures 1.23, 1.24). SetupEditor used without any arguments clears the stat list editor and restores the built-in lists L1-L6. $\diamond \square$

The TI-83 Plus Graphing Calculator Guidebook has much more information about lists, other ways they may be created and used. Most of the calculator programs presented in this supplement are illustrated using data stored in lists with descriptive names. Unless noted, you do *not* have to store data in the same descriptive-name or built-in list(s) used in the Calculator Solution. The programs normally display results on the Home Screen, and may temporarily use other named lists, built-in lists, or matrices, or change calculator modes.

Chapter 2

Descriptive Analysis and Presentation of Single–Variable Data

This chapter describes techniques for summarizing and describing data sets. There are four basic types of descriptive statistics: (1) measures of central tendency, (2) measures of dispersion (spread), (3) measures of position, and (4) types of distribution. The calculator may be used to produce graphical and numerical summaries, but emphasis here is also placed on interpretation of results.

2.1 Graphic Presentation of Data

A **circle graph**, or **pie diagram**, is used to summarize attribute data. It is a very common graph used in business settings, newspapers, and magazines to illustrate parts of the total. A circle is divided to show the amount of data that belongs to each category as a proportional part of the circle. The calculator program **CIRCLE**¹ may be used to construct a circle graph.

Example: The table below lists the number of cases of each type of operation performed at General Hospital last year. Display this data on a circle graph.

Calculator Solution:

(S1) On the Home Screen or using the stat list editor, enter the number of cases for each type of operation in the named list **OPERS** (Figure

¹Program by Chuck Vonder Embse, *Eightysomething*, Volume 3, Number 2, Spring 1994.

OPERS	L1	L2	1
20	-----	-----	
45			
58			
98			
115			
74			
65			
OPERS			
OPERS(B) = 23			

Figure 2.1: Number of cases for each type of operation.



Figure 2.2: Execute the program CIRCLE.

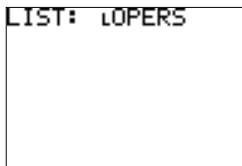


Figure 2.3: Input list containing data.



Figure 2.4: Options for displayed data.

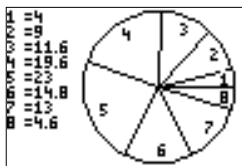


Figure 2.5: Circle graph for General Hospital data.

MAKE	NUMB	-----	1
1	75		
2	60		
3	35		
4	20		
5	5		
-----	-----		
MAKE(1) = 1			

Figure 2.6: Automobile make data.

2.1).

Note: As discussed in the Preface, descriptive-name or built-in lists may be used to store data. Named lists will be used in Calculator Solutions wherever possible to identify data and to illustrate more of the TI-83 Plus capabilities. \diamond

Type of Operation	Number of cases
Thoracic	20
Bones and joints	45
Eye, ear, nose, and throat	58
General	98
Abdominal	115
Urologic	74
Proctologic	65
Neurosurgery	23

- (S2) Choose the program CIRCLE from the **PRGM**; EXEC menu, and execute this program on the Home Screen (Figure 2.2).
- (S3) At the first program prompt, enter the name of the list containing the data, here OPERS (Figure 2.3). (Press **ENTER** after each input value.)

Note: A named list may be selected from the **LIST**; NAMES menu or entered from the keypad (preceded by the list indicator L). \diamond

- (S4) Select the method to display the data (along with the circle graph), percentages or actual data (Figure 2.4).
- (S5) The circle graph is displayed on the Graph Screen (Figure 2.5). The slices are given counterclockwise with the percentages (or data) in the top left corner of the screen. \square

A **bar graph** is often used to graphically summarize category or attribute data. A rectangle is drawn corresponding to each category, or class, with size proportional to the frequency. Bar charts are sometimes constructed so that the bars extend horizontally to the right. The TI-83 Plus may be used to construct a bar chart by assigning a number to each class or category. Use the statistical plot type histogram.

Example: The distribution of automobiles in the faculty parking lot on a recent Monday morning is given in the table below. Construct a bar graph that summarizes this categorical data.

Calculator Solution:

- (S1) Assign Ford 1, GM 2, Honda 3, Dodge 4, and Other 5. Enter the number for each make in the list MAKE and the corresponding count

in the list NUMB (Figure 2.6). Use the calculator to draw rectangles of width 1, centered at each class code.

Make	Number
Ford	75
GM	60
Honda	35
Dodge	20
Other	5

- (S2) Press **STATPLOT** and select Plot1 from the STATPLOTS menu.
- (S3) In the Plot1 menu select **On** and **Type** histogram. Set **Xlist** to the name of the list containing the code for each automobile make and **Freq** to the name of the list containing the number of each make (Figure 2.7). (Type the name of each list or select the name using **LIST**; NAMES.)
- (S4) In the **WINDOW**; WINDOW menu, enter values for the window variables so that the calculator will draw rectangles of width 1, centered at each class code. Since 1 is the midpoint of the first class, set **Xmin** to .5. Set **Xmax** to 5.5 and **Xscl** to 1 (the width of each class). **Ymin** should be 0 (the minimum number in any class), **Ymax** should be at least the largest count, and **Yscl** should be a reasonable distance between tick marks (Figure 2.8).
- (S5) Press **GRAPH** to display the bar chart (Figure 2.9).

Note:

- (N1) When displaying a bar chart, make sure there are no other **STATPLOTS** turned **On** and that there are no (selected) functions in the **Y=** editor. Otherwise the desired bar chart will be displayed simultaneously with any other selected graph.
- (N2) Use the **TRACE** key to move along the midpoint of each rectangle. The *class interval* and number are displayed at the bottom of the screen (Figure 2.10).
- (N3) The calculator bar chart is sometimes more pleasing if an extra empty class, or category, is added on each end of the graph. Figure 2.11 shows the new **WINDOW** settings and Figure 2.12 shows the resulting bar chart.
- (N4) A small space often separates each bar. This can be accomplished by changing the codes assigned to automobile make and/or modifying the viewing window. Figures 2.13 and 2.14 show an example with different values for **Xmin**, **Xmax**, and **Xscl**.
- (N5) It is possible to add text to a graph (and label the bars) by



Figure 2.7: Plot1 settings.

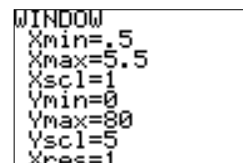


Figure 2.8: WINDOW settings.

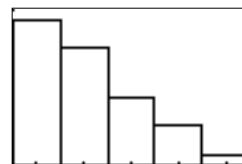


Figure 2.9: Bar chart for automobile data.

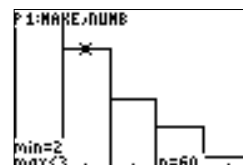


Figure 2.10: TRACE used with a STATPLOT; Type histogram.

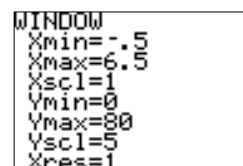


Figure 2.11: New WINDOW settings.

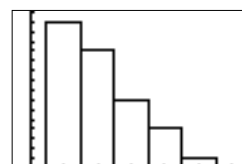


Figure 2.12: Bar chart with extra empty classes.

```

WINDOW
Xmin=.25
Xmax=6
Xsc1=.5
Ymin=0
Ymax=80
Ysc1=5
Xres=1

```

Figure 2.13: WINDOW settings to add extra space between bars.

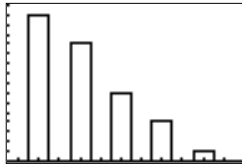


Figure 2.14: Bar chart with extra space

```

DEFEC ----- 1
R
12
56
23
45
6
-----
DEFEC(1)=8

```

Figure 2.15: Number of defects by category.

```

LIST: LDEFEC

```

Figure 2.16: Name of the list containing the defect numbers.

```

Ymax: 175
Ysc1: 25

```

Figure 2.17: Enter two WINDOW parameters.

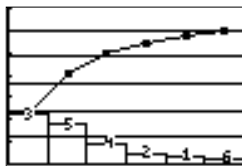


Figure 2.18: Pareto diagram.

using `DRAW`; `DRAW`; `Text`.

◇ □

A **Pareto Diagram** is a bar graph with the bars arranged from the most numerous category to the least numerous category. The diagram includes a line graph displaying the cumulative percentages and counts for the bars. The Pareto diagram is used often in quality-control applications to identify the numbers and types of defects that happen within a product or service. The calculator program `PARETO` may be used to display a Pareto diagram.

Example: The final daily inspection defect report for an assembly line at a local manufacturer is given in the table below.

Defect	Number
Dent	8
Bend	12
Blemish	56
Chip	23
Scratch	45
Others	6

Construct a Pareto diagram for this defect report. Management has given the production line the goal of reducing their defects by 50%. What two defects should they give special attention to in working toward this goal?

Calculator Solution:

- (S1) Enter the number of defects in the list `DEFEC` (Figure 2.15). Execute the program `PARETO`.
 - (S2) Enter the name of the list containing the number of defects (Figure 2.16), and values for `Ymax` and `Ysc1` (Figure 2.17). `Ymax` must be at least as big as the total number of defects. The Pareto diagram is displayed on the Graph Screen (Figure 2.18).
- Note:* The remaining `WINDOW` parameters are set by the calculator program. ◇
- (S3) The line graph connects the cumulative total defects and the horizontal lines are at 20, 40, 60, 80, and 100 percent of the total number of defects. The numbers at the top of each bar correspond to the complaint row in the original table.
 - (S4) Approximately 65% of the reported defects are due to blemishes and scratches. The assembly line crew should work to reduce these two defects in order to reach their goal. □

A **dotplot** is a graph used to display the distribution of a data set. The display represents each piece of data with a dot positioned along a mea-

surement scale. The measurement scale may be horizontal or vertical. The frequency of values is represented along the other scale. The calculator program DOTPLOT may be used to construct a dotplot.

Example: A random sample of 19 exam scores was selected from a large introductory statistics class. The scores are given in the table below.

Exam Scores									
76	74	82	96	66	76	78	72	52	68
86	84	62	76	78	92	82	74	88	

Construct a dotplot for this data.

Calculator Solution:

- (S1) Enter the exam score for each student in the list SCORE. Execute the program DOTPLOT.
- (S2) Enter the list containing the data (Figure 2.19). On the next input screen, enter values for the window parameters Xmin, Xmax, Xscl, and Ymax (Figure 2.20). The value for Ymax must be at least the largest frequency in any resulting dotplot column. The values for Ymin and Yscl are assigned by the program.
- (S3) The resulting dotplot is displayed on the Graph Screen (Figure 2.21).

Note:

- (N1) It may be necessary to execute DOTPLOT several times with different values for Xmin, Xmax, and Ymax in order to obtain an appropriate diagram.
- (N2) The value for Ymax must be less than or equal to 62. \diamond \square

A **frequency distribution** is a table that summarizes data by classes or class intervals. In a typical *grouped* frequency distribution, there are usually 5–20 classes of equal width. The table may contain columns for class number, class interval, tally (if constructing by hand), frequency, relative frequency, cumulative relative frequency, and class mark. In an *ungrouped* frequency distribution each class consists of a single value.

The calculator program FREQDIST constructs a frequency distribution that may be viewed in the stat list editor. Input includes the left-hand endpoint of the first class interval, the right-hand endpoint of the last class interval, and the width of each class interval.

Example: The hemoglobin A_{1c} test, a blood test given to diabetics during their periodic checkups, indicates the level of control of blood sugar during the past two to three months. The data in the table below was obtained

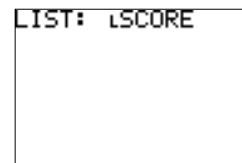


Figure 2.19: Enter the name of the list containing the data.

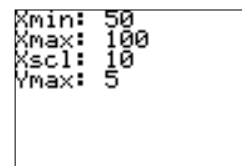


Figure 2.20: Enter values for the window parameters.

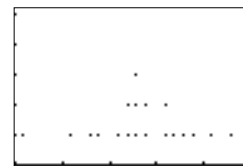


Figure 2.21: Dotplot for the exam scores.

for 40 different diabetics at a university clinic that treats diabetic patients. Construct a grouped frequency distribution using the classes $3.7 - <4.7$, $4.7 - <5.7$, $5.7 - <6.7$, etc. Which class has the highest frequency?

Blood Test Results

6.5	5.0	5.6	7.6	4.8	8.0	7.5	7.9	8.0	9.2
6.4	6.0	5.6	6.0	5.7	9.2	8.1	8.0	6.5	6.6
5.0	8.0	6.5	6.1	6.4	6.6	7.2	5.9	4.0	5.7
7.9	6.0	5.6	6.0	6.2	7.7	6.7	7.7	8.2	9.0

Calculator Solution:

- (S1) Enter the test result for each patient in the list BLOOD and execute the program FREQDIST. At the first program prompt enter the name of the list containing the data.
- (S2) Enter values for the lower class boundary of the frequency distribution (left-hand limit of the first class interval), upper class boundary (right-hand limit of the last class interval), and the width (for each class) (Figure 2.22).
- (S3) The frequency distribution is constructed and stored in the stat list editor. The first list contains the original data sorted in ascending order. The lists LBND and UBND identify each class interval, FREQ contains the frequency for each class, RELFQ contains the relative frequency for each class, and CUMRF contains the cumulative relative frequency for each class (Figures 2.23, 2.24).
- (S4) The class $5.7 - <6.7$ has the highest frequency. The frequency is 16 and the relative frequency is .40.

```

LW BOUND = 3.7
UP BOUND = 9.7
WIDTH = 1

```

Figure 2.22: FREQDIST input.

DATA	LBND	UBND	1
4.000	3.700	4.700	
4.800	4.700	5.700	
5.000	5.700	6.700	
5.000	6.700	7.700	
5.600	7.700	8.700	
5.600	8.700	9.700	
5.600	-----	-----	
DATA(1) = 4			

Figure 2.23: Sorted data and class intervals.

FREQ	RELFQ	CUMRF	4
4.000	.025	.025	
6.000	.150	.175	
16.000	.400	.575	
4.000	.100	.675	
10.000	.250	.925	
3.000	.075	1.000	
-----	-----	-----	
FREQ(1) = 1			

Figure 2.24: Frequency, relative frequency, and cumulative relative frequency.

Note:

- (N1) The program FREQDIST uses the TI-83 Plus built-in function **SetUpEditor** to store the frequency distribution in the stat list editor in the appropriate order. To remove the frequency distribution and restore the lists L1–L6, execute **SetUpEditor** without any arguments (lists) from the Home Screen.
- (N2) The calculator program places observations equal to a class boundary in the *larger* class. $\diamond \square$

A **histogram** is a bar graph representing a frequency distribution of a quantitative variable. The components of a histogram include:

- (H1) A title, which identifies the population or sample.
- (H2) A vertical scale, which identifies the frequencies in each class.
- (H3) A horizontal (measurement) scale, which identifies the variable. The class boundaries or class marks are labeled along the x axis.

The graph provides a quick look at the distribution of the data. A histogram is a built-in statistical plot on the TI-83 Plus.

Example: Construct a histogram for the blood test results given in the previous example.

Calculator Solution:

- (S1) Enter the data in the list BLOOD. Press **STATPLOT** and select Plot1 from the STATPLOTS menu.
- (S2) In the Plot1 menu select **On** and **Type** histogram. Set **Xlist** to the name of the list containing the data. (Type the name of the list or select the name using **LIST**; NAMES.) Set **Freq** to 1, the frequency of occurrence of each observation (Figure 2.25).
- (S3) In the **WINDOW**; WINDOW menu, select values for the window variables so that class intervals are evident and the entire graph fits on the calculator screen. Set **Xmin** to the lower class boundary of the first class, **Xmax** to the upper class boundary of the last class, and **Xscl** to the width of each class. **Ymin** should be 0 (the minimum frequency in any class), **Ymax** must be *at least* the largest frequency, and set **Yscl** to a reasonable distance between tick marks (Figure 2.26).
- (S4) Press **GRAPH** to display the histogram (Figure 2.27).

Note:

- (N1) When displaying a histogram, make certain there are no other STATPLOTS turned **On** and that there are no (selected) functions in the Y= editor. Otherwise the desired histogram will be displayed simultaneously with any other selected graph.
- (N2) To display a histogram on the TI-83 Plus, you do not need to construct a frequency distribution first. However, you must decide on the class intervals and widths, and you may have to experiment with values for **Ymax**.
- (N3) Use the **TRACE** key to move along the midpoint of each rectangle. The class interval and frequency are displayed at the bottom of the Graph Screen (Figure 2.28).
- (N4) The calculator histogram is sometimes more pleasing if an extra empty class is added on each end of the graph. Figure 2.29 shows the new WINDOW settings and Figure 2.30 shows the resulting graph. $\diamond \square$

Suppose a frequency distribution is given without the supporting data. It is still possible to use the TI-83 Plus to construct a histogram based on the class marks.

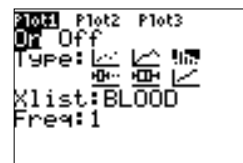


Figure 2.25: Plot1 menu.

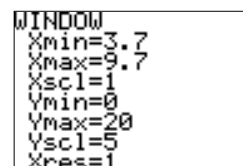


Figure 2.26: WINDOW settings.

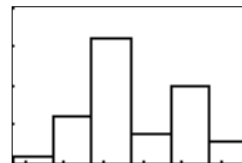


Figure 2.27: Histogram display.

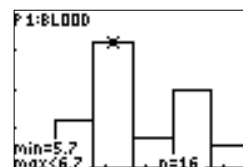


Figure 2.28: TRACE used with a histogram.

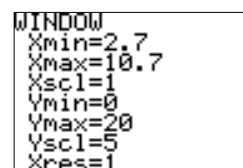


Figure 2.29: New WINDOW settings.

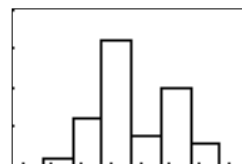


Figure 2.30: Histogram with extra empty classes.

AGE	COUNT	-----	1
25	34		
35	58		
45	76		
55	187		
65	254		
75	241		
85	147		

Figure 2.31: Class marks and frequencies.

Plot1	Plot2	Plot3
On	Off	
Type:		
Xlist: AGE		
Freq: COUNT		

Figure 2.32: Plot1 settings.

WINDOW
Xmin=10
Xmax=100
Xscl=10
Ymin=0
Ymax=275
Yscl=25
Xres=1

Figure 2.33: WINDOW settings.

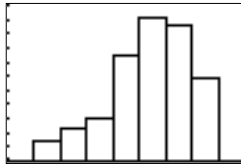


Figure 2.34: Histogram for age data.

Example: A recent survey of Roman Catholic nuns summarized their ages in the table below.

Age	Frequency	Class Mark
20 up to 30	34	25
30 up to 40	58	35
40 up to 50	76	45
50 up to 60	187	55
60 up to 70	254	65
70 up to 80	241	75
80 up to 90	147	85

Construct a histogram for this age data.

Calculator Solution:

- (S1) Enter the class mark for each interval in the list `AGE` and the frequency for each interval in the corresponding row of the list `COUNT` (Figure 2.31).
- (S2) In the `Plot1` menu, select `On`, `Type` histogram, `Xlist:AGE`, and set `Freq` to the list containing the class frequencies, `COUNT` (Figure 2.32).
- (S3) Select appropriate `WINDOW` settings (Figure 2.33). Press `GRAPH` to display the histogram (Figure 2.34). \square

An **ogive** is a plot of cumulative frequency or cumulative relative frequency versus class limit. A horizontal scale identifies the upper class boundaries. Every ogive starts on the left with a (relative) frequency of zero at the lower class boundary of the first class and ends on the right with a cumulative relative frequency of 1, or cumulative frequency of n (the number of observations in the data set).

Example: An insurance company recently surveyed employees to determine their annual salary. The frequency distribution below gives the results for 50 random employees in \$1000.

Class Boundaries	Frequency	Cumulative Frequency	Cumulative Relative Frequency
$35 \leq x < 45$	2	2	0.04
$45 \leq x < 55$	2	4	0.08
$55 \leq x < 65$	7	11	0.22
$65 \leq x < 75$	13	24	0.48
$75 \leq x < 85$	11	35	0.70
$85 \leq x < 95$	11	46	0.92
$95 \leq x < 105$	4	50	1.00

Construct an ogive using cumulative relative frequencies for this employee salary data.

Calculator Solution:

- (S1) Enter the class boundaries in the list `SALRY` and the corresponding cumulative relative frequencies in the list `CUMRF` (Figure 2.35).
- (S2) Select appropriate settings for `Plot1` (Figure 2.36) and `WINDOW` (Figure 2.37). Press `GRAPH` to view the ogive (Figure 2.38). \square

SALRY	CUMRF	----	1
25,000	0.0000		
45,000	.0400		
55,000	.0800		
65,000	.2200		
75,000	.4800		
85,000	.7000		
95,000	.9200		
105,000	1.0000		
SALRY(0) = 105			

Figure 2.35: Class boundaries and cumulative relative frequencies.

2.2 Numerical Descriptive Statistics

A measure of central tendency is a numerical value, or single number statistic, that locates the middle of a data set. It represents a *typical* observation or indicates where the majority of the data is clustered.

Four common measures of central tendency are defined below.

- (D1) The **(arithmetic) mean**, \bar{x} , of a set of values is the sum of the values divided by the total number of values.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{n}(x_1 + x_2 + \cdots + x_n)$$

- (D2) The **median**, \tilde{x} , of a set of values is the value in the middle position when the data is ranked in order according to size.
- (D3) The **mode** is the value that appears most frequently. If two or more values in a sample are tied for the highest frequency, there is no mode.
- (D4) The **midrange** is the number exactly midway between the lowest value L and the highest value H : $\text{midrange} = (L + H)/2$.

Example: A recent *Atlantic Monthly* (Nov. 1990) contained the number of days in the standard school year for several different countries. The data is given in the table below.

School Days								
216	190	195	185	182	160	233	216	200
200	192	180	185	190	192	200	211	175
180	185	180	190	184	180	191	243	220

Find the mean and the median number of days per year of school for the countries given.

Plot1	Plot2	Plot3
On	Off	Off
Type: L	Eq	dh
Xlist: SALRY		
Ylist: CUMRF		
Mark: \square	+	.

Figure 2.36: Plot1 settings.

WINDOW
Xmin=25
Xmax=115
Xscl=10
Ymin=0
Ymax=1.1
Yscl=.1
Xres=1

Figure 2.37: WINDOW settings.

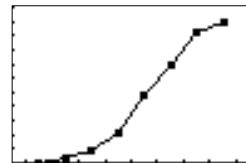


Figure 2.38: Ogive for salary data.

```

1-Var Stats LDAY
S

```

Figure 2.39: Built-in function 1-Var Stats.

```

1-Var Stats
x̄=194.63
Σx=5255.00
Σx²=1031585.00
Sx=18.40
σx=18.06
n=27.00

minX=160.00
Q1=182.00
Med=190.00
Q3=200.00
maxX=243.00

```

Figure 2.40: Summary statistics.

```

mean(LDAYS) 194.63
median(LDAYS) 190.00

```

Figure 2.41: Mean and median list functions.

```

LIST: LMIN5

```

Figure 2.42: Input list containing data.

```

CENTRAL TENDENCY
1: MIDRANGE
2: MODE
3: EXIT

```

Figure 2.43: Program menu.

```

MIDRANGE = 29.50

```

Figure 2.44: Computed midrange.

Calculator Solution:

- (S1) Enter the data in the list `DAYS`.
- (S2) Use the TI-83 Plus built-in function for summary statistics to find the mean and median: `[STAT]`; `CALC`; `1-Var Stats` (Figure 2.39, 2.40). The mean is 194.63 and the median is 190.00.

Note:

- (N1) If a list is not given as an argument, `1-Var Stats` uses the data in the list `L1` by default.
- (N2) The mean and/or median may also be found on the Home Screen using built-in functions in the `[LIST]`; `MATH` menu (Figure 2.41).

The mode and midrange may be computed using the program `CENTRAL`.

Example: Recruits for a police academy were required to undergo a test that measures their exercise capacity. The exercise capacity (measured in minutes) was obtained for each of 20 recruits and is given in the table below.

Exercise Capacity									
25	27	30	33	30	32	30	34	30	27
26	25	29	31	31	32	34	32	33	30

Find the mode and midrange.

Calculator Solution:

- (S1) Enter the exercise capacity data in the list `MINS`. Execute the program `CENTRAL`. At the first program prompt enter the name of the list containing the data (Figure 2.42).

- (S2) Select the `MIDRANGE` option from the program menu (Figure 2.43). The midrange (29.50) is displayed on the Home Screen (Figure 2.44).

Note: After displaying a measure of central tendency, the program `CENTRAL` is paused. (The pause indicator is in the top-right corner of the screen. To continue execution, press `[ENTER]`.) ◇

- (S3) Press `[ENTER]` to return to the program menu. Select option 2 to compute the mode. The mode is displayed on the Home Screen and the program pauses (Figure 2.45). The mode is 30 and it occurs 5 times.

- (S4) Press `[ENTER]` to return to the program menu. Select option 3 to exit the program.

Note: If there is no mode, the program will find and display the

smallest value among those values with the tied, highest frequency.

◇ □

Measures of central tendency describe the *middle* of a data set. These statistics describe only one aspect of the data. Two very *different* data sets may have similar measures of central tendency. Measures of dispersion indicate *variability*, or spread, and are also used to characterize a data set. Some common measures of dispersion are defined below.

Suppose x_1, x_2, \dots, x_n is a set of observations.

(D1) The **range** is the difference in value between the highest value and the lowest value.

(D2) The **sample variance** is the sum of the squared deviations about the mean divided by $n - 1$:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2.$$

The **sample standard deviation** is the positive square root of the sample variance: $s = \sqrt{s^2}$.

Each of these measures may be computed on the Home Screen using an arithmetic expression and/or built-in calculator functions.

Example: A small business wanted to know the daily amount each of its employees spent for lunch. A random sample of 20 employees revealed the following amounts.

Lunch Money						
13.95	15.85	4.50	6.75	12.00	8.25	15.75
4.15	1.25	14.10	10.75	15.00	8.15	22.70
2.45	17.80	14.05	7.50	10.65	4.10	

Find the range, variance, and standard deviation.

Calculator Solution:

(S1) Enter the data in the list LUNCH. Compute 1-Var Stats for the data in this list. The standard deviation, Sx, is given in the top output screen: $s = 5.71$ (Figure 2.46).

Note: After executing 1-Var Stats, the computed summary statistics are displayed on the Home Screen and stored in calculator variables. These variables may be used in arithmetic expressions in order to find other summary statistics. ◇

(S2) In the **[VARS]**; VARS; Statistics; XY menu use the calculator variables that stand for the highest value and the lowest value, maxX and

```
MODE =
COUNT = 30.00
5.00
```

Figure 2.45: Computed mode and frequency.

```
1-Var Stats
x=10.49
Σx=209.70
Σx²=2818.36
Sx=5.71
σx=5.57
n=20.00

minX=1.25
Q1=5.63
Med=10.70
Q3=14.55
maxX=22.70
```

Figure 2.46: Standard deviation, highest value, and lowest value computed using 1-Var Stats.

```

maxX-minX    21.45
Sx²          32.61

```

Figure 2.47: Range and variance computed using calculator variables.

```

max(LUNCH)-min(LUNCH)    21.45

```

Figure 2.48: Range computed using `LIST`; MATH functions.

```

Variance(LUNCH)    32.61
stdDev(LUNCH)     5.71

```

Figure 2.49: Variance and standard deviation computed using `LIST`; MATH functions.

`minX`, in an arithmetic expression to find the range (Figure 2.47).

- (S3) In the `VAR`; `VAR`; `Statistics`; `XY` menu use the variable that stands for sample variance, `Sx`, in an arithmetic expression to find the variance (Figure 2.47).

Note: These summary measures of dispersion may be computed on the Home Screen using `LIST`; MATH operations (Figures 2.48, 2.49). $\diamond \square$

If data is presented in the form of an ungrouped frequency distribution, the formulas below may be used to find the mean, variance, and standard deviation. If the data is presented in the form of a grouped frequency distribution, the class marks may be used in the formulas below in order to *estimate* the mean, variance, and standard deviation.

Let x_1, x_2, \dots, x_k be a set of values (in an ungrouped frequency distribution) or a set of class marks (in a grouped frequency distribution).

Suppose f_1, f_2, \dots, f_k are the corresponding frequencies and $n = \sum_{i=1}^k f_i$.

- (D1) The mean for the frequency distribution is given by

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum x_i f_i}{n}$$

- (D2) The variance for the frequency distribution is given by

$$s^2 = \frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{\sum f_i}}{\sum f_i - 1} = \frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{n}}{n - 1}$$

The standard deviation for the frequency distribution is $s = \sqrt{s^2}$.

These statistics may be computed using the calculator program `GROUPED`.

Example: A farmer conducted an experiment in order to judge the value of a new diet for his animals. The weight gain (in grams) for chicks fed on a high-protein diet is given in the table below.

Weight Gain	Frequency
12.5	2
12.7	6
13.0	22
13.1	29
13.2	12
13.8	4

Find the mean, variance, and standard deviation.

Calculator Solution:

- (S1) The data is given in the form of an ungrouped frequency distribution. Enter the value for each weight gain in the list **GAIN** and the frequency for each gain in the corresponding row of the list **FREQS** (Figure 2.50).
- (S2) Execute the program **GROUPED**. Input the name of the list containing the marks or data, and the name of the list containing the frequencies (Figure 2.51).
- (S3) The results are displayed on the Home Screen. The mean is 13.0760, $s^2 = .0532$, and $s = .2307$ (Figure 2.52). \square

GAIN	FREQS	----	1
12.50	2.00		
12.70	6.00		
13.00	22.00		
13.10	29.00		
13.20	12.00		
13.80	4.00		
-----	-----		
GAIN(1) =			12.5

Figure 2.50: Data and frequencies in the stat list editor.

MARKS:	LGAIN
FREQS:	LFREQS

Figure 2.51: Input lists.

MEAN =	13.0760
VARIANCE =	.0532
STD DEVIATION =	.2307
	Done

Figure 2.52: Resulting summary statistics for the frequency distribution.

Measures of position are used to locate a specific value in relation to the rest of the data. **Quartiles** divide the ranked data into four parts. The first quartile, Q_1 , is a number such that at most 25% of the data is smaller in value than Q_1 and at most 75% is larger. The second quartile is the median. The third quartile, Q_3 , is a number such that at most 75% of the data is smaller in value than Q_3 and at most 25% is larger.

Percentiles divide a set of ranked data into 100 equal subsets; each set of data has 99 percentiles. The k th percentile, P_k , is a value such that at most $k\%$ of the data is smaller in value than P_k and at most $(100 - k)\%$ of the data is larger.

Two additional measures of central tendency are defined in terms of the quartiles. The **midquartile** is the numerical value midway between the first quartile and the third quartile: $\text{midquartile} = (Q_1 + Q_3)/2$. The **interquartile range** is the difference between the first and third quartiles: $\text{iqr} = Q_3 - Q_1$.

These four measures of position may be computed using the calculator program **POSITION**.

Example: An experiment was conducted in order to test how quickly certain fabrics ignite when exposed to a flame. The table below lists the ignition times for a synthetic fabric.

Ignition Times								
30.1	31.5	34.0	37.5	30.1	31.6	34.5	37.5	30.2
31.6	34.5	37.6	30.5	32.0	35.0	38.0	31.0	32.4
35.0	39.5	31.1	32.5	35.6	31.2	33.0	36.0	31.3
33.0	36.5	31.3	33.0	36.9	31.4	33.5	37.0	

Find the quartiles, the midquartile, the interquartile range, and the 88th percentile.

```

STATISTICS
1:QUARTILES
2:PERCENTILE
3:MIDQUARTILE
4:IQR
5:EXIT

```

Figure 2.53: Main program menu.

```

Q1 =      31.3000
Q3 =      36.0000

```

Figure 2.54: Quartiles.

```

MIDQUARTILE =
              33.6500

```

Figure 2.55: Midquartile.

```

IQR =      4.7000

```

Figure 2.56: Interquartile range.

```

PERCENTILE: 88

```

Figure 2.57: Input for k th percentile.

```

KTH PERCENTILE =
                37.5000

```

Figure 2.58: 88th percentile.

Calculator Solution:

- (S1) Enter the ignition time data in the list `IGNIT`. Execute the program `POSITION`.
- (S2) At the first program prompt, enter the name of the list containing the data. Select the `QUARTILES` option from the program menu (Figure 2.53). The quartiles are displayed on the Home Screen and the calculator pauses (Figure 2.54). $Q_1 = 31.3$ and $Q_3 = 36.0$. Press `ENTER` to return to the program menu.
- (S3) Select the `MIDQUARTILE` option and then the `IQR` option from the main menu to find the midquartile and interquartile range (Figure 2.55, 2.56). The midquartile is 33.65 and the interquartile range is 4.7.
- (S4) Select the `PERCENTILE` option from the program menu and input the value for k (Figure 2.57). The k th percentile is displayed on the Home Screen and the calculator pauses (Figure 2.58). $P_{88} = 37.5$. Press `ENTER` to return to the program menu and select `EXIT` to quit the program.

Note: The quartiles, midquartile, and interquartile range may be computed on the Home Screen using built-in calculator functions and/or arithmetic expressions.

- (N1) Compute `1-Var Stats` for the data in the list `IGNIT`. The quartiles are displayed in the bottom screen of output (Figure 2.59) and all displayed values are stored in calculator statistics variables.
- (N2) In the `[VARS]; VARS; Statistics; PTS` menu use the calculator variables that stand for the first and third quartile, Q_1 and Q_3 , in an arithmetic expression to find the midquartile and the interquartile range (Figure 2.60). $\diamond \square$

The 5-number summary is an effective way to describe a set of data and is composed of:

- (1) L , the smallest value in the data set,
- (2) Q_1 , the first quartile (also called P_{25} , the 25th percentile),
- (3) \tilde{x} , the median,
- (4) Q_3 , the third quartile (also called P_{75} , the 75th percentile),
- (5) H , the largest value in the data set.

A **box-and-whisker display**, or boxplot, is a graphic representation of the 5-number summary. The five numerical values (smallest, first quartile, median, third quartile, and largest) are located on a scale, either horizon-

tal or vertical. A box is drawn with edges at the quartiles and a line is drawn at the median. A line segment (whisker) is drawn from Q_1 to the smallest value, and another line segment is drawn from Q_3 to the largest value. This regular box-and-whisker display is a built-in statistical plot.

The TI-83 Plus will also display a modified box-and-whisker display. This display uses the inner fences defined by

$$IF = Q_1 - (1.5 * iqr), \quad IF = Q_3 + (1.5 * iqr)$$

to identify outliers. Each whisker is drawn to the most extreme value within the inner fence. Any value beyond an inner fence is marked with \square , $+$, or \cdot .

Example: A research study of manual dexterity involved determining the time required to complete a certain task. The time (in seconds) required for each of 40 first graders is given in the table below.

Dexterity Test Times									
7.1	8.3	9.4	11.0	7.2	8.4	9.6	11.1	7.2	8.4
9.9	11.2	7.6	8.9	10.1	11.2	7.6	9.0	10.1	11.2
7.9	9.0	10.1	12.0	8.1	9.1	10.2	13.6	8.1	9.1
10.3	14.7	8.1	9.1	10.5	14.9	8.3	9.1	10.7	15.5

Find the 5-number summary and draw the box-and-whisker display.

Calculator Solution:

- (S1) Enter the dexterity test times in the list **TIMES**. Compute the **1-Var Stats** for this list. The 5-number summary is displayed in the bottom screen of output (Figure 2.61): L (**minX**) = 7.10, Q_1 = 8.30, \tilde{x} (**Med**) = 9.25, Q_3 = 10.85, and H (**maxX**) = 15.50.
- (S2) In the **STATPLOT** menu, select **Plot1**. In the **Plot1** menu, select **On** and **Type** regular boxplot (no outliers), set **Xlist** to **TIMES** and **Freq** to 1 (Figure 2.62).
- (S3) Select appropriate **WINDOW** settings (Figure 2.63) and press **GRAPH** to display the boxplot (Figure 2.64).

Note:

- (N1) The TI-83 Plus ignores the y axis **WINDOW** settings when drawing a boxplot.
- (N2) Up to three boxplots may be displayed simultaneously. This makes it easy to graphically compare different samples. If only one plot is displayed, it is always placed in the top third of the Graph Screen, regardless of plot number.

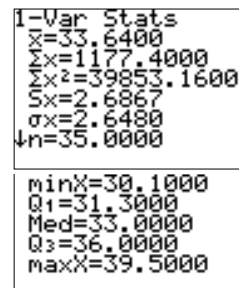


Figure 2.59: Quartiles computed using 1-Var Stats.

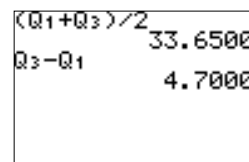


Figure 2.60: Midquartile and interquartile range computed using statistics variables.

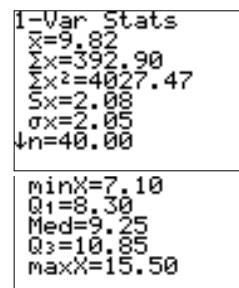


Figure 2.61: 1-Var Stats for **TIMES**.

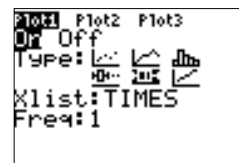


Figure 2.62: **Plot1** settings for regular boxplot.

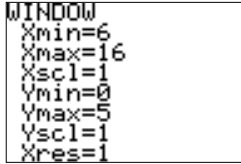


Figure 2.63: WINDOW settings.

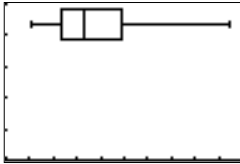


Figure 2.64: Regular boxplot for data in TIMES.

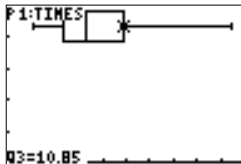


Figure 2.65: Use of TRACE with a boxplot.

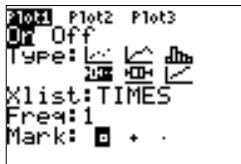


Figure 2.66: Plot1 settings for a modified boxplot.

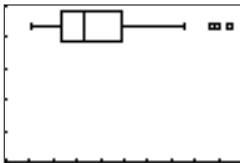


Figure 2.67: Modified boxplot for data in TIMES.



Figure 2.68: z-score for 65.

- (N3) Press **TRACE** to move along the 5-number summary for the boxplot (Figure 2.65).
- (N4) To display a modified boxplot, in the **Plot1** menu change **Type** to the fourth graph (boxplot with outliers), and select a **Mark** for any outliers (Figure 2.66). Press **GRAPH** to display the modified boxplot (Figure 2.67). There are three outliers, values beyond an inner fence. $\diamond \square$

The **z-score**, or **standard score**, for a specific value is a measure of relative standing in terms of the mean and standard deviation. The z-score for a value x is

$$z = \frac{\text{value} - \text{mean}}{\text{st. dev.}} = \frac{x - \bar{x}}{s}.$$

The z-score may be found using TI-83 Plus built-in functions and/or arithmetic expressions.

Example: All applicants for a certain civil service position are required to take a special aptitude test. Since the pool of applicants varies considerably from year to year, new employees are often hired on the basis of the relative standing of their test score. The scores from fifteen randomly selected applicants are given in the table below. Find the z-score for the (specific) value 65.

Aptitude Test Scores							
35	55	25	75	80	50	45	65
70	40	30	45	30	55	70	

Calculator Solution:

- (S1) Enter the data in the list **APT**.
- (S2) On the Home Screen, enter an arithmetic expression using **LIST**; **MATH** functions to find the z-score for 65 (Figure 2.68). 65 is .77 standard deviations away from the mean.

Note: Use the list **APT** in the arithmetic expression to find the z-score for each observation (Figure 2.69). Figure 2.70 shows each observation along with its z-score. $\diamond \square$

The following result allows us to interpret and use the standard deviation to describe a sample or population, and to compare the variability of one data set with another.

Chebyshev's Theorem: The proportion of any distribution that lies within k standard deviations of the mean is at least $1 - (1/k^2)$, where k is any positive number larger than 1. This theorem applies to all distributions of data.

The calculator program CHEBY applies this theorem to a sample or population, summary statistics or a set of observations.

Example: A recent article compared algebra courses that used computer-assisted instruction with traditional lecture courses. The scores that the computer-assisted group made on an achievement test consisting of 50 problems had these summary statistics: $n = 57$, $\bar{x} = 23.14$, $s = 7.02$. Find the limits within which at least 75% of the scores fall.

Calculator Solution:

- (S1) Execute the program CHEBY. Since in this example the summary statistics are known, select **STATS** at the first program prompt (Figure 2.71).

Note: Use the **LIST** option if the sample data or the entire population is stored in a list. \diamond

- (S2) Enter values for the mean, standard deviation, and k (Figure 2.72).
- (S3) At least 75% of the scores fall in the interval (9.10, 37.18) (Figure 2.73). Press **ENTER** to return to the **K:** prompt. The program will continue to generate results for other values of k , or for a value less than or equal to 1 the program will exit. \square

```
(LAPT-mean(LAPT)
)/stdDev(LAPT)*Z
SCOR
(-.92 .21 -1.49...
```

Figure 2.69: Calculation of each z-score.

AAPT	ZSCOR	-----	1
85.00	-.92	-----	
55.00	.21	-----	
25.00	-1.49	-----	
75.00	1.34	-----	
80.00	1.62	-----	
50.00	-.08	-----	
45.00	-.36	-----	
65.00	.77	-----	
70.00	1.06	-----	
40.00	-.64	-----	
30.00	-1.21	-----	
45.00	-.36	-----	
30.00	-1.21	-----	
55.00	.21	-----	
70.00	1.06	-----	

Figure 2.70: Each value and z-score.

```
INPUT:
1:LIST
2:STATS
```

Figure 2.71: Summary statistics known.

```
MEAN: 23.14
SDEV: 7.02
K: 2
```

Figure 2.72: Input values for \bar{x} , s , and k .

```
K =
2.00
AT LEAST
.75
WITHIN
9.10
37.18
```

Figure 2.73: Resulting proportion and interval.

Notes:

Chapter 3

Descriptive Analysis and Presentation of Bivariate Data

When bivariate data results from two qualitative (attribute or categorical) variables, the data is often arranged on a *cross-tabulation* or *contingency table*. The calculator program CROSSTAB may be used to summarize data in a cross-tabulation and to obtain row, column, and total percentages.

Example: Thirty students from a college were randomly identified and classified according to two variables: (1) gender (M/F) and (2) major (liberal arts, business administration, technology), as shown in the table below.

Student	Gender	Major	Student	Gender	Major
1	M	LA	16	M	BA
2	F	BA	17	M	LA
3	M	LA	18	M	BA
4	F	LA	19	F	LA
5	M	BA	20	M	T
6	M	T	21	M	BA
7	F	LA	22	F	BA
8	M	T	23	M	T
9	F	BA	24	F	LA
10	F	BA	25	M	T
11	M	T	26	M	BA
12	M	LA	27	F	LA
13	F	LA	28	F	T
14	M	T	29	M	BA
15	F	T	30	M	LA

Summarize this data in a 2×3 cross-tabulation table.

Calculator Solution:

GEND	MAJOR	-----	1
1	1		
2	1		
1	1		
1	1		
1	1		
1	1		
2	1		

GEND(1) = 1

Figure 3.1: Coded data for the first seven students.

```

ROWS: LGEND
COLS: LMAJOR
    
```

Figure 3.2: Input lists.

```

[A]
[[15 6 7 18]
 [6 4 2 12]
 [11 10 9 30]]
    
```

Figure 3.3: Cross-tabulation table, frequencies.

```

[B]
[[28 33 39 100]
 [50 33 17 100]
 [37 33 30 100]]
    
```

Figure 3.4: Cross-tabulation table, row percentages.

```

[C]
...45 60 78 60...
...55 40 22 40...
...100 100 100 10...
    
```

Figure 3.5: Cross-tabulation table, column percentages.

- (S1) Code the data as follows: for gender M = 1, F = 2; for major LA = 1, BA = 2, and T = 3. Enter the gender value for each student in the named list GEND and the corresponding major values in the list MAJOR. Figure 3.1 shows the coded data for the first seven students.
- (S2) Execute the program CROSSTAB. Input the name of the list corresponding to rows and the list corresponding to columns in the cross-tabulation table (Figure 3.2).
- (S3) The cross-tabulation table with the frequency for each cross category of the two variables along with marginal totals is stored in the matrix [A]. The cross-tabulation table showing row percentages is stored in the matrix [B], column percentages in the matrix [C], and overall percentages based on the grand total in the matrix [D]. All of these percentage matrices also contain marginal totals.
- (S4) Each matrix may be viewed on the Home Screen or in the matrix editor (MATRIX; EDIT). Figure 3.3 shows the matrix [A], frequency for each category, on the Home Screen. Here is the interpretation of the entries in the matrix [A].

		Major			Row Totals
		LA	BA	T	
Gender	Male	5	6	7	18
	Female	6	4	2	12
Col Totals		11	10	9	30

- (S5) Figure 3.4 shows the cross-tabulation table of percentages based on row totals and Figure 3.5 shows the percentages based on column totals (both displayed on the Home Screen).
- (S6) Figure 3.6 shows the cross-tabulation table of percentages of grand total, displayed in the matrix editor. The table indicates 60% of the sample were male, 40% female, 36.7% were Liberal Arts majors, etc.

```

MATRIX[D] 3 x4 3 x4
[[16.7 20.0 23.3 60.0 ]
 [20.0 13.3 6.7 40.0 ]
 [36.7 33.3 30.0 100.0 ]
    
```

Figure 3.6: Cross-tabulation table, percentages.

Note: The coded data for each list must begin with 1 and use only consecutive positive integers. ◇ □

The cross-tabulation tables like those constructed in the Example above may be displayed in side-by-side bar graphs using the calculator program **GROUPBAR**.

Example: Construct a bar graph displaying the row percentages for the gender-major data given in the Example above.

Calculator Solution:

- (S1) Using the cross-tabulation table for the row percentages (Figure 3.4), enter the percentages for each major, by gender, in the lists **LA**, **BA**, and **T** (Figure 3.7). Execute the program **GROUPBAR**.
- (S2) On the program menu, select the number of groups (only 2 or 3) to display side-by-side (Figure 3.8). In this example there are three groups, or majors.
- (S3) Enter the three lists containing the percentages, a reasonable value for **Ymax** (the maximum percentage), and a value for **Yscl** (Figure 3.9).
- (S4) The bar chart is displayed on the graph screen (Figure 3.10). Unshaded bars correspond to the first major (**LA**), horizontal shaded bars correspond to the second major (**BA**), and diagonal shaded bars correspond to the third major (**T**). □

If bivariate data consists of one qualitative and one quantitative variable, the quantitative values may be viewed as belonging to separate samples. Each sample, or data set, is identified by levels of the qualitative variable. Using the TI-83 Plus, single-variable summary statistics (**1-Var Stats**) may be obtained for each sample and the results may be displayed in tabular form for comparison. The TI-83 Plus may also be used to display up to three boxplots simultaneously, on the same measurement axis, in order to compare the distribution of each sample.

Example: The long distance charges for randomly selected phone numbers using one of three different carriers are given in the table below.

Company A		Company B		Company C	
20.30	23.20	6.95	9.80	38.40	23.75
19.80	28.95	14.75	13.75	55.40	24.80
22.70	27.40	9.25	11.80	38.80	42.00
24.50	24.60	12.15	21.05	16.75	24.40
21.70	28.40	13.15	13.60	33.05	16.00

Construct a boxplot for each sample and display the graphs side-by-side.

LA	BA	T	1
REF0	33.0	39.0	
50.0	33.0	17.0	

LAC1=28			

Figure 3.7: Percentages for each major, by gender.

NO. OF GROUPS
1:2
2:3

Figure 3.8: Number of groups.

LIST1:	LLA
LIST2:	LBA
LIST3:	LT
Ymax:	55
Yscl:	10

Figure 3.9: Input lists and WINDOW parameters.

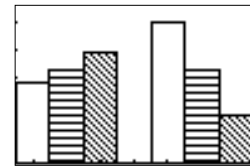


Figure 3.10: Bar graph.

COMP A	COMP B	COMP C
20.20	6.95	28.40
19.80	14.75	35.40
22.70	9.25	38.80
24.50	12.15	16.75
21.70	13.15	33.05
23.20	9.80	33.75
28.95	13.75	24.80
22.40	11.80	42.00
24.60	21.05	24.40
28.40	13.60	16.00

COMPAB) = 27.4

Figure 3.11: Data for each phone company.



Figure 3.12: Plot1 settings.

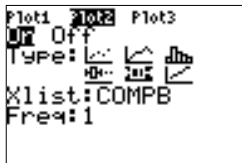


Figure 3.13: Plot2 settings.



Figure 3.14: Plot3 settings.

CALOR	FAT	----- 1
420.00	9.00	
210.00	20.00	
450.00	10.00	
130.00	6.00	
310.00	25.00	
290.00	7.00	
450.00	20.00	
446.00	20.00	
640.00	38.00	
233.00	11.00	
552.00	55.00	
360.00	6.00	
838.00	20.00	
493.00	12.00	
360.00	28.00	
345.00	28.00	
552.00	22.00	

CALOR(15) = 199

Figure 3.17: Calorie and Fat data.

Compare the distributions.

Calculator Solution:

- (S1) Enter the data for each company in a separate list: COMP A, COMP B, and COMP C (Figure 3.11).
- (S2) Select appropriate plot settings for each of the three statistical plots (Figures 3.12, 3.13, 3.14) and WINDOW parameters (Figure 3.15).
- (S3) Press **GRAPH** to display the boxplots simultaneously (Figure 3.16).

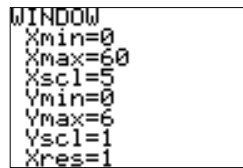


Figure 3.15: WINDOW settings.

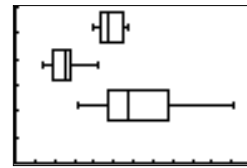


Figure 3.16: Simultaneous boxplots.

Both Company A and Company B have compact distributions, in comparison with Company C. Most of the data for Company B is smaller than Companies A and C. The data for Company C has more variability and larger values. □

If bivariate data consists of two quantitative variables, the values are often expressed mathematically as ordered pairs (x, y) . The first value, x , is the input, or independent, variable, and the second value, y , is the output, or dependent, variable. This bivariate data may be presented graphically on a **scatter diagram**: a plot of all ordered pairs of bivariate data on a coordinate axis system. The input variable x is plotted on the horizontal axis, and the output variable y is plotted on the vertical axis. A scatter diagram, or scatter plot, is a TI-83 Plus built-in statistical plot.

Example: A recent study investigated the calories and fat in some popular fast-food items. The data is given in the table below.

Calories (x)	270	420	210	450	130	310	290	450	446
Fat (y)	9	20	10	22	6	25	7	20	20
Calories (x)	640	233	552	360	838	199	360	345	552
Fat (y)	38	11	55	6	20	12	36	28	22

Construct a scatter diagram for this data. Is there any pattern?

Calculator Solution:

- (S1) Enter the Calorie data in the list CALOR and the Fat data in the list FAT (Figure 3.17).

- (S2) Set up Plot1 (**STATPLOT**; STATPLOTS; Plot1). Select On, Type scatter plot (the first plot type), set Xlist to CALOR, Ylist to FAT, and select the desired Mark (Figure 3.18).
- (S3) Select appropriate WINDOW settings (Figure 3.19) and press **GRAPH** to display the scatter diagram (Figure 3.20).
- (S4) The scatter diagram suggests the two variables are related. As the number of calories rise, so does the fat content.

Note:

- (N1) Use the **TRACE** key to move along the points on the graph. The x and y values are displayed at the bottom of the screen (Figure 3.21).
- (N2) A quick method to find a reasonable viewing window is to use ZoomStat (in the **ZOOM**; ZOOM menu) first. This will automatically select WINDOW settings so that all data points will be displayed on the Graph Screen. The WINDOW settings usually need to be modified to include more *friendly* numbers. $\diamond \square$

Suppose $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ represent n pairs of observations on two variables. The **coefficient of linear correlation**, or **Pearson's product moment correlation coefficient**, r is a numerical measure of the strength of the linear relationship between the two variables. The formula for r is

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{(n - 1)s_x s_y} = \frac{SS(xy)}{\sqrt{SS(x)}\sqrt{SS(y)}}.$$

Note:

- (N1) The summation in the formula for r extends from $i = 1$ to n .
- (N2) s_x and s_y are the standard deviations of the x and y variables.
- (N3) $SS(x), SS(y), SS(xy)$ are the sum of squares for x, y , and xy , respectively.
- (N4) $-1 \leq r \leq +1$: Coefficients of $+1$ and -1 describe perfect linear correlation.
- (N5) The closer r is to 1 in either direction, the greater the strength of the correlation. The strength of the correlation is not dependent on the direction.
- (N6) The **coefficient of determination**, denoted r^2 , is the proportion of the total variation in one variable that is explained by the other variable. It is computed by squaring the coefficient of linear correlation, $r^2 = r \cdot r$. The magnitude of the coefficient of determination is smaller than that of the coefficient of linear correlation. It is a



Figure 3.18: Plot1 settings.

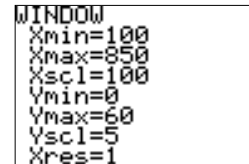


Figure 3.19: WINDOW settings.

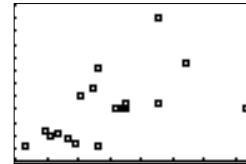


Figure 3.20: Scatter diagram.

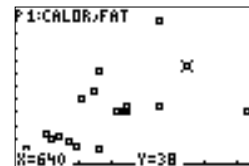


Figure 3.21: **TRACE** feature with a scatter diagram.



Figure 3.22: Plot1 settings.

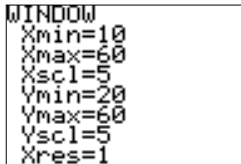


Figure 3.23: WINDOW settings.

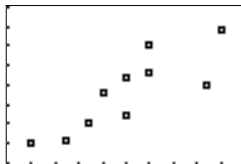


Figure 3.24: Scatter diagram.

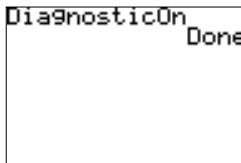


Figure 3.25: Execute DiagnosticOn to display r and r^2 .

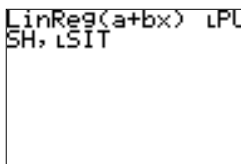


Figure 3.26: Use of LinReg(a+bx) to compute r and r^2 .

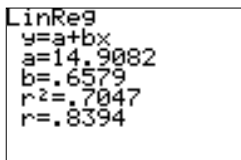


Figure 3.27: LinReg(a+bx) results.

more conservative measure of the linear relationship between two variables. \diamond

Example: In a certain physical education class, several fitness scores were recorded. The following sample is the number of push-ups and sit-ups done by ten randomly selected students.

Student	1	2	3	4	5	6	7	8	9	10
Push-ups (x)	27	22	15	35	30	52	35	55	40	40
Sit-ups (y)	30	26	25	42	38	40	32	54	50	43

Construct a scatter diagram and compute the coefficient of linear correlation.

Calculator Solution:

- (S1) Enter the push-up and sit-up data in the lists PUSH and SIT, respectively.
- (S2) Set up Plot1 (**STATPLOT**; STATPLOTS; Plot1). Select On, Type scatter plot, set Xlist to PUSH, Ylist to SIT, and select the desired Mark (Figure 3.22).
- (S3) Select appropriate WINDOW settings (Figure 3.23) and press **GRAPH** to display the scatter diagram (Figure 3.24).
- (S4) On the Home Screen select DiagnosticOn from the **CATALOG** menu. Press **ENTER** to execute this command (Figure 3.25).

Note: The coefficient of linear correlation and coefficient of determination are computed using the function LinReg(a+bx) (or the function LinReg(ax+b)) but displayed only if DiagnosticOn is in effect. If DiagnosticOff is used, r and r^2 will not be displayed. \diamond

- (S5) On the Home Screen, select LinReg(a+bx) (in the **STAT**; CALC menu), and enter the lists containing the observations on the independent variable (x) and the dependent variable (y) (Figure 3.26). Press **ENTER** to execute this calculator function.
- (S6) The results are displayed on the Home Screen (Figure 3.27). $r = .8394$ and $r^2 = .7047$. The scatter diagram and the coefficient of linear correlation indicate a strong positive correlation between push-ups and sit-ups for students in the physical education class.

Note: The function LinReg(a+bx) also computes and displays the coefficients for the regression line (a and b). $\diamond \square$

Regression analysis is used to find the equation of the line that best describes the relationship between two variables. The equation for the *best* line is called the **regression equation** and it may be used to pre-

dict a value of the dependent variable based on a selected value of the independent variable.

The calculator may be used to employ the **method of least squares** in order to find the *best-fitting* line. The calculation of the coefficients of the regression line and a hypothesis for a significant regression are built-in calculator functions. Using optional function arguments and plotting features it is possible to view the graph of the regression line and the scatter diagram on the same set of axes, and to obtain predicted values.

Example: In a random sample of eight college women, each was asked for her height (to the nearest inch) and her weight (to the nearest five pounds). The data is given in the following table.

	1	2	3	4	5	6	7	8
Height (x)	65	65	62	67	69	65	61	67
Weight (y)	105	125	110	120	140	135	95	130

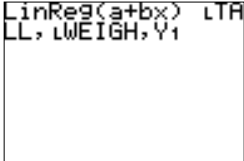
- (E1) Find an equation to predict the weight of a college woman based on her height (the equation of the line of best fit).
- (E2) Construct a graph of the regression line and the scatter diagram.
- (E3) Determine the coefficient of linear correlation and the coefficient of determination.
- (E4) If a college woman is 65 inches, what is her predicted weight?

Calculator Solution:

- (S1) Enter the height and weight for each woman in the lists TALL and WEIGH, respectively.
- (S2) Select the function **LinReg(a+bx)** (in the **STAT**; **CALC** menu) and enter three arguments: independent variable (TALL), dependent variable (WEIGH), and a function variable (Y1, in the **VAR**S; **Y-VARS**; **Function** menu) for storing the regression equation (Figure 3.28). Press **ENTER** to execute this command.

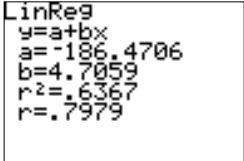
Note:

- (N1) The third argument in **LinReg(a+bx)** (calculator function variable) is optional. The regression equation is always stored in the statistics variable **RegEQ**.
- (N2) Execute **DiagnosticOn** before **LinReg(a+bx)** so that r and r^2 will be displayed. ◇
- (S3) The slope and the y -intercept for the equation of the line of best fit, r , and r^2 are displayed on the Home Screen (Figure 3.29). The regression equation is $\hat{y} = -186.4706 + 4.7059x$, $r = .7979$ and $r^2 = .6367$.



```
LinReg(a+bx) LTA
LL, WEIGH, Y1
```

Figure 3.28:
LinReg(a+bx) function
and arguments.



```
LinReg
y=a+bx
a=-186.4706
b=4.7059
r^2=.6367
r=.7979
```

Figure 3.29:
LinReg(a+bx) results.



Figure 3.30: Plot1 settings.

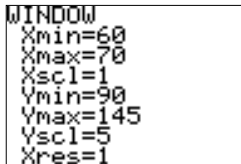


Figure 3.31: WINDOW settings.

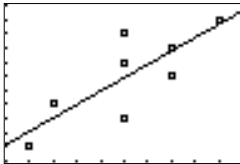


Figure 3.32: Scatter diagram and regression line.

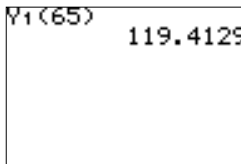


Figure 3.33: Predicted weight for 65 inches.

- (S4) Select Plot1 settings (Figure 3.30) and appropriate WINDOW settings (Figure 3.31). Press **GRAPH** to display the scatter diagram and the graph of the equation of the line of best fit (Figure 3.32).

Note: Since the equation of the line of best fit was stored in the function variable Y1, the graph of this line will automatically be displayed (along with the scatter diagram). ◇

- (S5) In order to compute the predicted value, evaluate Y1 at 65 (Figure 3.33). The equation of the line of best fit predicts a weight of 119.4129 for a college woman 65 inches tall. □

Chapter 4

Probability

In a sample space containing sample points that are equally likely to occur, the probability $P(A)$ of an event A is the ratio of the number $n(A)$ of points that satisfy the definition of event A to the number $n(S)$ of sample points in the entire sample space:

$$P(A) = \frac{n(A)}{n(S)}.$$

In an equally likely outcome experiment, determining the probability of an event A is often reduced to counting possibilities.

The **Fundamental Counting Rule** and the **General Counting Rule** are basic *counting* techniques that follow from constructing a tree diagram that shows all possible outcomes in an experiment.

A **permutation** is an ordered arrangement of a set of distinct objects. That is, there is a first object, a second object, a third object, and so on; and each object is distinctly different from the others. The permutation formula is

$${}_n P_r = n \times (n - 1) \times (n - 2) \times \cdots \times (n - r + 1) = \frac{n!}{(n - r)!}$$

where n is the total number of distinct objects and r is the number of different objects selected. The TI-83 Plus has a built-in function for finding permutations.

Example: A group of eight finalists in a ceramic art competition are to be awarded five prizes; first through fifth. Three of the finalists win nothing. How may different ways are there to award these five prizes?

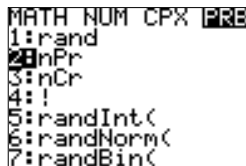


Figure 4.1: **MATH**; PRB menu.

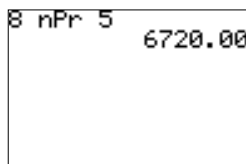


Figure 4.2: sP_5 .

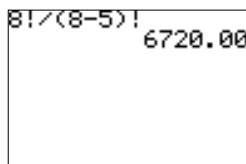


Figure 4.3: Permutations computed using the factorial symbol.

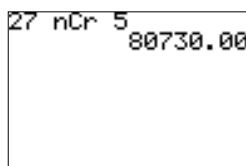


Figure 4.4: Combination computed using nCr .

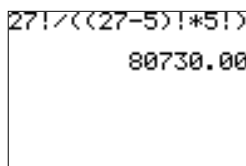


Figure 4.5: Combination computed using the factorial symbol.

Calculator Solution:

- (S1) Since the prizes are ordered, the answer to this example involves a permutation. Enter 8 on the Home Screen. Press **MATH** and select PRB; nPr (Figure 4.1).
- (S2) Enter 5 and press **ENTER**. The number of different ways of awarding the five prizes is 6720 (Figure 4.2).

Note:

- (N1) The TI-83 Plus function nPr accepts two arguments as input: one *before* and one *after* the token. This is different from the common idea for function and from most other TI-83 Plus functions.
- (N2) Permutations may also be computed on the Home Screen by using the factorial symbol (!) (Figure 4.3). This symbol is also in the **MATH**; PRB menu. However, using the factorial symbol may lead to overflow or round-off errors. $\diamond \square$

A **combination** is a set of distinct objects without regard to an arrangement or an order. The membership of the set is all that matters. The combination formula is

$${}_nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} = \frac{n!}{(n-r)!r!} = \frac{{}_nP_r}{r!}$$

where n is the total number of distinct objects and r is the number of objects selected. The TI-83 Plus has a built-in function for finding combinations.

Example: The Statistics Department at a large university has 27 members and a committee of 5 is needed to review tenure applications. How many different possible committees are there?

Calculator Solution:

- (S1) Since the order or the arrangement of the committee members is not important, the solution involves a combination. On the Home Screen, enter the number of distinct Department members, 27.
- (S2) Select **MATH**; PRB; nCr , and enter 5. The number of different committees of 5 is 80730 (Figure 4.4).

Note: Combinations may also be computed directly on the Home Screen using the factorial symbol (Figure 4.5). $\diamond \square$

Example: A Statistics Department has 30 members and a committee is needed to review promotion applications. The committee is to be composed of two co-chairpersons and three members. How many different

possible committees are there?

Calculator Solution:

- (S1) The problem is solved by considering two parts: selecting two co-chairpersons and then the remaining committee members. Use the Fundamental Counting Rule ($m \times n$) where m is the number of possible choices for the co-chairpersons and n is the number of three-person committees.
- (S2) The number of different promotion committees of size 5 with assigned co-chairpersons is

$$n(\text{committees}) = m \times n = {}_{30}C_2 \times {}_{28}C_3 = 1425060.$$

See Figure 4.6.

Note: This product may also be computed directly on the Home Screen (Figure 4.7). $\diamond \square$

Example: A small business has just receive shipment of 24 computers. Suppose 4 of the machines are actually defective and that 6 machines will be given to the Accounting Department.

- (E1) How many different groups of 6 machines can the Accounting Department receive?
- (E2) Find the probability the Accounting Department will receive exactly one defective machine.
- (E3) Find the probability the Accounting Department will receive at least 3 defective machines.

Calculator Solution:

- (S1) Since the order of selecting the computers is not important, the solution involves a combination. The number of different groups of 6 computers selected from 24 is ${}_{24}C_6 = 134596$ (Figure 4.8).
- (S2) Since this is an equally likely outcome experiment, the probability of selecting one defective computer (and 5 good computers) is

$$P(1 \text{ defective}) = \frac{{}_4C_1({}_{20}C_5)}{{}_{24}C_6} = \frac{4 \cdot 15504}{134596} = .4608$$

See Figure 4.9.

- (S3) The probability of selecting at least 3 defective computers is the probability of selecting 3 or 4 defectives.

$$P(\text{at least 3 defectives}) = \frac{{}_4C_3({}_{20}C_3)}{{}_{24}C_6} + \frac{{}_4C_4({}_{20}C_2)}{{}_{24}C_6} = .0353$$

See Figure 4.10. \square

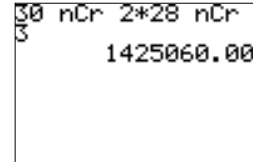


Figure 4.6: Product of combinations.

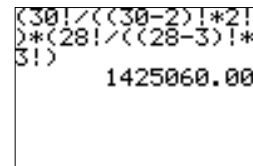


Figure 4.7: Product of combinations using the factorial symbol.

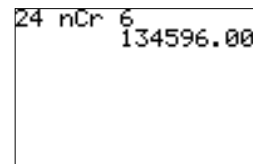


Figure 4.8: ${}_{24}C_6$.

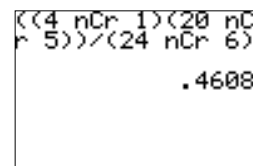


Figure 4.9: Probability of selecting 1 defective.

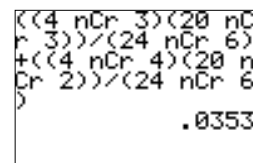


Figure 4.10: probability of selecting at least 3 defectives.

Bayes' Rule is an expanded form for conditional probabilities. This rule may be used to compute *reversed* conditional probabilities from given conditional and unconditional probabilities. Suppose A_1, A_2, \dots, A_n is a collection of nonoverlapping events such that the union of these events is the entire sample space, $P(A_i) \neq 0, i = 1, 2, \dots, n$. For any event B with $P(B) \neq 0$

$$P(A_k | B) = \frac{P(A_k \cap B)}{P(B)} = \frac{P(A_k) \cdot P(B | A_k)}{\sum_{i=1}^n P(A_i) \cdot P(B | A_i)}$$

The program **BAYES** applies this rule to a collection of conditional and unconditional probabilities.

Example: A company that manufactures shoes has three factories. Factory 1 produces 25% of the company's shoes, Factory 2 produces 60%, and Factory 3 produces 15%. One percent of the shoes produced by Factory 1 are mislabeled, 0.5% of those produced by Factory 2 are mislabeled, and 3% of those produced by Factory 3 are mislabeled. Suppose one pair of shoes manufactured by this company is selected at random.

- (E1) Find the probability the shoes are mislabeled.
- (E2) Suppose the shoes are mislabeled. Find the probability they were manufactured at Factory 2.

Calculator Solution:

- (S1) The three Factories make up the nonoverlapping events. Let A_i be the event Factory i manufactures the shoes. Enter the *unconditional* probabilities $P(A_i)$ in list **FACTS** (Figures 4.11).
- (S2) Let B stand for the event of mislabeling a pair of shoes. The *known* conditional probabilities are $P(B | A_i)$. Enter these probabilities in the list **MISL** (Figure 4.11).
- (S3) Execute the program **BAYES**. Enter the list containing the unconditional probabilities at the first prompt and the list containing the known conditional probabilities at the second prompt (Figure 4.12).
- (S4) Probabilities associated with Bayes' Rule are computed and placed in the stat list editor. View the results using **[STAT]**; **EDIT**; **EDIT** (Figure 4.13).

FACTS	MISL	-----	1
.2500	.0100		
.6000	.0050		
.1500	.0300		
-----	-----		
FACTS(1) = .25			

Figure 4.11: Known unconditional and conditional probabilities.

```
UNCOND: LFACTS
COND: LMISL
```

Figure 4.12: **BAYES** input.

Here is an explanation of each list.

```
UNCON: P(Ai)
COND: P(B | Ai)
INTER: P(Ai ∩ B)
BAYES: P(Ai | B)
PRB: P(B) (first row only)
```

From the stat list editor, the probability of a mislabeled pair of shoes

is $P(B) = .01$. Given a pair of mislabeled shoes, the probability is was manufactured at Factory 2 is $P(A_2 | B) = .30$. \square

UNCON	COND	INTER 1	BAYES	PRE 5
.2500	.0100	.0025	.2500	.0100
.6000	.0050	.0030	.3000	-----
.1500	.0300	.0045	.4500	-----
-----	-----	-----	-----	-----
UNCON(1) = .25			.01	

Figure 4.13: Probabilities associated with Bayes' Rule.

Notes:

Chapter 5

Probability Distributions (Discrete Variables)

A **random variable** is a function that assumes a unique numerical value for each of the outcomes in the sample space of a probability experiment. A random variable is **discrete** if it can assume a finite number or countably infinite number of values. A random variable is **continuous** if its set of possible values is an entire interval of numbers.

The **probability distribution** for a discrete random variable X is a distribution of the probabilities associated with each of the values of the random variable. The **probability function**, or **probability mass function** (pmf), for X is the rule that assigns probabilities to the values of the random variable.

Suppose X is a discrete random variable with probability function $P(x)$. The **mean**, or **expected value** of X is given by

$$\mu = E(X) = \sum_{\text{all } x} x_i \cdot P(x_i) = \sum [xP(x)].$$

The **variance** of X is given by

$$\sigma^2 = E[(X - \mu)^2] = \sum_{\text{all } x} (x_i - \mu)^2 \cdot P(x_i) = \sum [(x - \mu)^2 P(x)].$$

The **standard deviation** of X is the positive square root of the variance:

$$\sigma = \sqrt{\sigma^2}.$$

The calculator program **DISCRV** may be used to compute these three summary measures for a discrete random variable.

VALS	PROBS	-----	1
10.00	.40		
11.00	.20		
12.00	.20		
13.00	.10		
14.00	.10		
-----	-----		
VALS(1) = 10			

Figure 5.1: Values and probabilities for X .

VALUES:	LVALS
PROBS:	LPROBS

Figure 5.2: Input lists.

MEAN =	11.30
VARIANCE =	1.81
STD DEV =	1.35
	Done

Figure 5.3: Mean, variance, and standard deviation for X .

Example: The number of ships to arrive at a harbor on any given day is a random variable with probability distribution given below. Find the mean, variance, and standard deviation for X .

x	10	11	12	13	14
$P(x)$	0.4	0.2	0.2	0.1	0.1

Calculator Solution:

- (S1) This is an example involving a discrete random variable because the number of values X assumes is finite. Enter the values for X in the list **VALS** and the corresponding probabilities in the list **PROBS** (Figure 5.1).
- (S2) Execute the program **DISCRV**. Enter the list containing the values and the list containing the probabilities (Figure 5.2).
- (S3) The mean number of ships arriving is $\mu = 11.3$ with variance $\sigma^2 = 1.81$ and standard deviation $\sigma = 1.35$ (Figure 5.3). \square

A **binomial probability experiment** consists of repeated trials that possess the following properties:

- (P1) There are n repeated independent trials.
 (P2) Each trial has two possible outcomes (success, failure).
 (P3) $P(\text{success}) = p$, $P(\text{failure}) = q$, and $p + q = 1$.

A **binomial random variable** is a discrete random variable that is the count of the number of successful trials that occur in a binomial experiment. If X is a binomial random variable with n trials and probability of a success p , the **binomial probability function** is

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x}, \quad \text{for } x = 0, 1, 2, \dots, n.$$

Some probability questions concerning a binomial random variable may be solved using tables. More general problems may be solved using the TI-83 Plus built-in functions for the probability function (**binomialpdf**) and cumulative probability function (**binomialcdf**). Both are in the **DISTR**; **DISTR** menu. Every question concerning a binomial random variable may be answered using cumulative probability. However, in some cases using the *probability* function requires fewer calculations or keystrokes.

Example: A recent survey suggests 60% of all adults drink coffee in the morning. Six adults are randomly selected and asked if they drink coffee in the morning. The random variable X is the number of adults who respond yes. Construct the probability distribution for the random variable X .

Calculator Solution:

- (S1) The general form of the calculator binomial probability function is `binomialpdf(n, p, x)`. If a value for x is omitted, the function returns a list of probabilities for all possible values of the random variable. The first row (in the resulting list) corresponds to $X = 0$, the second row to $X = 1$, etc.
- (S2) In order to make the probability distribution easier to interpret, first enter the numbers 0–6 in the list X. Use the probability function to find the probabilities and store the resulting list in PROBS (Figure 5.4).
- (S3) The probability distribution may be viewed in the stat list editor (Figure 5.5). For example, the probability that 4 out of 6 sampled adults drink coffee in the morning is .3110.

Note: The TI-83 Plus may also be used to create a table of binomial cumulative probabilities and store the results in a list. See Figures 5.6 and 5.7 ◇ □

Example: There are many reasons for mortgage foreclosures. According to a recent financial survey, disability causes 48% of all mortgage foreclosures. Suppose 25 random mortgage foreclosures are audited by a large lending institution. Find the probability of the following events.

- (E1) Exactly 10 foreclosures are due to disability.
 (E2) At least 14 are due to disability.
 (E3) At most 12 are due to disability.
 (E4) Between 9 and 15 (inclusive) are due to disability.

Calculator Solution:

- (S1) Assume the properties of a binomial probability experiment hold. Let X be the number of mortgage foreclosures due to disability out of the 25 selected. X is a binomial random variable with $n = 25$ and $p = .48$. Use the probability function or cumulative probability function to answer each question.
- (S2) $P(X = 10) = P(X \leq 10) - P(X \leq 9) = .1166$ (Figure 5.8).
- (S3) $P(X \geq 14) = 1 - P(X \leq 13) = .2740$ (Figure 5.9).
- (S4) $P(X \leq 12) = .5801$ (Figure 5.10).
- (S5) $P(9 \leq X \leq 15) = P(X \leq 15) - P(X \leq 8) = .8401$ (Figure 5.11). □

A **Poisson random variable** is often used to model the number of successes in a fixed interval of time or within a specified region. The Poisson distribution is used frequently to model rare events. If X is a Poisson random variable with mean μ , the probability distribution for X is given

```
binomialpdf(6,.6)→P
PROBS
(.0041 .0369 .1...
```

Figure 5.4: Binomial probabilities computed on the Home Screen and stored in a list.

X	PROBS	-----	1
0.0000	.0041		
1.0000	.0369		
2.0000	.1382		
3.0000	.2765		
4.0000	.3110		
5.0000	.1866		
6.0000	.0467		

Figure 5.5: Binomial probability distribution in the stat list editor.

```
binomcdf(6,.6)→C
PROB
(.0041 .0410 .1...
```

Figure 5.6: Cumulative probabilities computed on the Home Screen and stored in a list.

X	PROBS	CPROB	1
0.0000	.0041	.0041	
1.0000	.0369	.0410	
2.0000	.1382	.1792	
3.0000	.2765	.4557	
4.0000	.3110	.7667	
5.0000	.1866	.9533	
6.0000	.0467	1.0000	

Figure 5.7: Values of X , probabilities, and cumulative probabilities.

```
binomcdf(25,.48,
10)-binomcdf(25,
.48,9)
.1166
binomialpdf(25,.48,
10)
.1166
```

Figure 5.8: $P(X = 10)$.

```
1-binomcdf(25,.4
8,13)
.2740
```

Figure 5.9: $P(X \geq 14) = 1 - P(X \leq 13)$.

```
binomcdf(25,.48,
12)
.5801
```

Figure 5.10: $P(X \leq 12)$

```
binomcdf(25,.48,
15)-binomcdf(25,
.48,8)
.8401
```

Figure 5.11:
 $P(9 \leq X \leq 15) =$
 $P(X \leq 15) - P(X \leq 8)$.

```
PoissonPdf(5,LX)
→PROBS
(.0067 .0337 .0...
```

Figure 5.12: Poisson probabilities computed on the Home Screen and stored in a list.

X	PROBS	----- 1
0.0000	.0067	
1.0000	.0337	
2.0000	.0842	
3.0000	.1404	
4.0000	.1755	
5.0000	.1755	
6.0000	.1462	

Figure 5.13: First few rows of a Poisson probability distribution in the stat list editor.

by

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}, \quad \text{for } x = 0, 1, 2, 3, \dots$$

Problems concerning a Poisson random variable may be answered using the TI-83 Plus built-in functions `poissonpdf` (probability function) and `poissoncdf` (cumulative probability function). Both are in the **DISTR**; DISTR menu.

Example: The number of people entering the emergency ward at a certain hospital during any 24 hour period is a Poisson random variable X with mean 5. Construct the first few rows of the probability distribution for X .

Calculator Solution:

- (S1) The general form of the calculator Poisson probability function is `poissonpdf` (μ, x). The second argument must be included but may be a calculator list.
- (S2) Enter the numbers 0–6 in the list **X**. Use the probability function to find the probabilities and store the results in the list **PROBS** (Figure 5.12).
- (S3) The first few rows of the probability distribution for X may be viewed in the stat list editor (Figure 5.13). For example, $P(X = 3) = .1404$. \square

In repeated, independent trials, suppose the probability of a success is p and the probability of a failure is $q = 1 - p$. The probability distribution for the **geometric random variable** X , the trial number on which the first success occurs, is given by

$$P(x) = pq^{x-1}, \quad x = 1, 2, 3, \dots$$

Problems concerning a geometric random variable may be answered using the TI-83 Plus built-in functions `geometpdf` (probability function) and `geometcdf` (cumulative probability function). Both are in the **DISTR**; DISTR menu.

Example: In a special promotion, a fast food restaurant claims 1 out of every 20 game pieces wins a prize. Suppose game pieces are selected at random. Find the probability of the following events.

- (E1) The first winning piece is selected on the eighth try.
 (E2) At least 10 pieces must be selected before the first winner is found.
 (E3) The first winning piece is found in 4 or less tries.

Calculator Solution:

- (S1) Let X be the the trial (game piece) on which the first success (winner) occurs. X is a geometric random variable with $p = 1/20 =$

.05. Use the probability function or cumulative probability function to answer each question. The calculator geometric probability and cumulative probability functions take two arguments, the probability of a success and a value of X : `geometpdf(p, x)` and `geometcdf(p, x)`.

(S2) $P(X = 8) = P(X \leq 8) - P(X \leq 7) = .0349$ (Figure 5.14).

(S3) $P(X \geq 10) = 1 - P(X \leq 9) = .6302$ (Figure 5.15).

(S4) $P(X \leq 4) = .1855$ (Figure 5.16). □

```

geometcdf(.05,8)
-geometcdf(.05,7)
)
.0349
geometpdf(.05,8)
.0349

```

Figure 5.14: $P(X = 9)$.

```

1-geometcdf(.05,
9)
.6302

```

Figure 5.15: $P(X \geq 10) = 1 - P(X \leq 9)$.

```

geometcdf(.05,4)
.1855

```

Figure 5.16: $P(X \leq 4)$.

Notes:

Chapter 6

Normal Probability Distributions

A continuous random variable assumes values in an entire interval of real numbers. The **probability distribution function** (or probability density function, pdf), $f(x)$, for a continuous random variable X is defined such that the probability that X takes on any value between a and b ($a \leq b$) is the area under the curve $f(x)$, above the x axis, and between the lines $x = a$ and $x = b$.

The **cumulative distribution function** (cdf) for a continuous random variable X is defined as

$$F(x) = P(X \leq x),$$

and geometrically it is the area under the pdf to the left of x . The cdf for a continuous random variable X may be used to answer any probability question concerning X .

A **normal random variable** X has as its probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2}$$

where μ is the mean ($-\infty < \mu < \infty$), σ is the standard deviation ($\sigma > 0$) and x can assume any real number. A **standard normal random variable**, Z , has mean 0 ($\mu = 0$) and standard deviation 1 ($\sigma = 1$).

Traditionally, probability questions concerning a normal random variable are answered by standardizing. The probability statement is first transformed, or converted, into an equivalent problem in the Z world. The standard normal cumulative distribution function is then used to obtain the desired probability. Standardization is *not* always necessary before

```

Plot1 Plot2 Plot3
\Y1=normalpdf(X,
40,3)
\Y2=normalpdf(X,
40,8)
\Y3=
\Y4=
\Y5=

```

Figure 6.1: Normal probability distribution functions.

```

WINDOW
Xmin=15
Xmax=65
Xscl=5
Ymin=0
Ymax=.15
Yscl=.05
Xres=1

```

Figure 6.2: WINDOW settings.

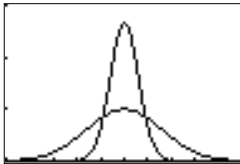


Figure 6.3: Graphs of normal probability distribution functions.

using the TI-83 Plus to help solve a normal probability question. There are four built-in functions in the **DISTR** menu to help solve problems and illustrate concepts concerning a normal probability distribution.

Example: Use the TI-83 Plus to illustrate the normal probability distributions with (1) $\mu = 40$, $\sigma = 3$ and (2) $\mu = 40$, $\sigma = 8$.

Calculator Solution:

- (S1) The TI-83 Plus function `normalpdf` (in the **DISTR**; **DISTR** menu) is used to evaluate a normal probability distribution function at any value x . It takes three arguments, x , μ , and σ . If μ and σ are omitted, then the distribution is assumed to be standard normal.
- (S2) In the **Y=** editor, enter the two probability distribution functions (Figure 6.1).
- (S3) Select appropriate **WINDOW** settings (Figure 6.2).
- (S4) Press **GRAPH** to display the probability distribution functions (Figure 6.3). The taller, more compact graph has $\sigma = 3$.

Note: The *style* of either graph may be changed in the **Y=** editor in order to better identify each probability distribution. $\diamond \square$

Example: Assume Z is a standard normal random variable. Find the probability that Z is

- (E1) greater than 1.35.
- (E2) less than -1.35 .
- (E3) between -1.4 and 1.4 .

Calculator Solution:

- (S1) Use the built-in function `normalcdf` (in the **DISTR**; **DISTR** menu) to answer each question. This function takes four arguments: a , b , μ , and σ and returns the probability that a normal random variable with mean μ and standard deviation σ is between a and b . If μ and σ are omitted, the distribution is assumed to be standard normal. Wherever possible, two calculator solutions will be presented: the first using the standard normal cumulative distribution function, and the second without rewriting the problem in terms of cumulative probability.

```

1-normalcdf(-1E9
9,1.35)
normalcdf(1.35,1
E99)
.0885
.0885

```

Figure 6.4: $P(Z > 1.35) = 1 - P(Z \leq 1.35)$.

- (S2) $P(Z > 1.35) = 1 - P(Z \leq 1.35) = .0885$ (Figure 6.4).

Note:

- (N1) $-1E99$ is considered the smallest number the TI-83 Plus can work with, and $1E99$ is the largest, effectively $-\infty$ and ∞ .
- (N2) The calculator function `ShadeNorm` (in the **DISTR**; **DRAW** menu)

may be used to illustrate this probability, or area. This function takes four arguments: a , b , μ and σ . It draws a normal probability distribution function with mean μ and standard deviation σ , shades the region between a and b , and computes the area under the curve between a and b . If μ and σ are omitted, a standard normal distribution is assumed. Figure 6.5 shows the WINDOW settings and Figure 6.6 illustrates the use of `ShadeNorm` on the Home Screen. Figure 6.7 shows the resulting graph and calculation. \diamond

(S3) $P(Z < -.52) = .3015$ (Figure 6.8).

(S4) $P(-1.4 < Z < 1.4) = P(Z < 1.4) - P(Z \leq -1.4) = .8385$ (Figure 6.9). \square

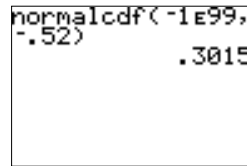


Figure 6.8:
 $P(Z < -.52)$.

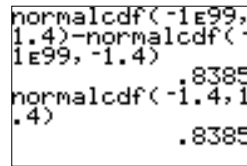


Figure 6.9:
 $P(-1.4 < Z < 1.4) =$
 $P(Z < 1.4) -$
 $P(Z \leq -1.4)$.

Example: What is the 75th percentile for a standard normal random variable?

Calculator Solution:

(S1) Find the value of a such that $P(Z < a) = .75$.

(S2) Use the function `invNorm`, located in the `DISTR`; `DISTR` menu. This function takes three arguments: p , μ , and σ . It returns a value x such that $P(X < x) = p$, where X is a normal random variable with mean μ and standard deviation σ . If μ and σ are omitted, a standard normal distribution is assumed.

(S3) Figure 6.10 shows the 75th percentile for a standard normal distribution is .6745.

Note: Figure 6.11 illustrates the solution to this problem using `ShadeNorm`. \diamond \square

Example: For a particular age group of adult males, the distribution of cholesterol readings, in mg/dl, is normally distributed with a mean of 210 and a standard deviation of 15. Suppose an adult male in this age group is selected at random. What is the probability his cholesterol reading will be

(E1) less than 201?

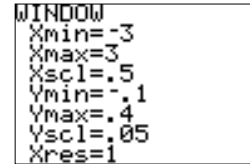


Figure 6.5: WINDOW settings.



Figure 6.6: `ShadeNorm` used to illustrate probability.

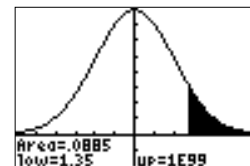


Figure 6.7: `ShadeNorm` graph and calculation.

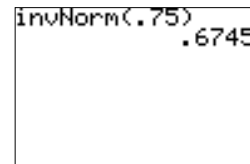


Figure 6.10: Inverse cumulative probability function for a standard normal random variable.

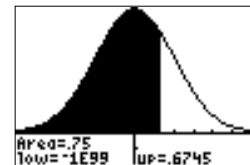


Figure 6.11: `ShadeNorm` graph and calculation to confirm and illustrate the solution.

```
normalcdf(-1E99,
-.6)
.2743
normalcdf(-1E99,
201,210,15)
.2743
```

Figure 6.12:
P($X < 201$).

```
1-normalcdf(-1E99,
.4)
.3446
normalcdf(216,1E99,
210,15)
.3446
```

Figure 6.13:
P($X > 216$).

```
normalcdf(-1E99,
1.4)-normalcdf(-1E99,
-.2)
.4985
normalcdf(207,231,
210,15)
.4985
```

Figure 6.14:
P($207 < X < 231$).

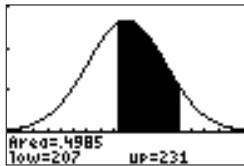


Figure 6.15: ShadeNorm graph and calculation.

```
invNorm(.95,3.7,
1.4)
6.0028
```

Figure 6.16: invNorm with $\mu = 3.7$, $\sigma = 1.4$.

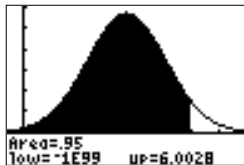


Figure 6.17: ShadeNorm illustrates the solution.

- (E2) more than 216?
(E3) between 207 and 231?

Calculator Solution:

- (S1) Use `normalcdf` to answer each question. Wherever possible, two calculator solutions will be presented: the first by standardizing and then using the standard normal cumulative probability function, and the second without standardizing or rewriting the problem in terms of cumulative probability.

$$(S2) P(X < 201) = P\left(\frac{X - 210}{15} < \frac{201 - 210}{15}\right) = P(Z < -.6) = .2743$$

(Figure 6.12).

$$(S3) P(X > 216) = P\left(\frac{X - 210}{15} > \frac{216 - 210}{15}\right) = P(Z > .4)$$

$$= 1 - P(Z \leq .4) = .3446 \text{ (Figure 6.13).}$$

$$(S4) P(207 < X < 231) = P\left(\frac{207 - 210}{15} < \frac{X - 210}{15} < \frac{231 - 210}{15}\right)$$

$$= P(-.2 < Z < 1.4) = P(Z < 1.4) - P(Z \leq -.2)$$

$$= .4985 \text{ (Figure 6.14).}$$

Note: Figure 6.15 illustrates this probability using `ShadeNorm`. $\diamond \square$

Example: The waiting time for a teller at a certain bank is approximately normally distributed with a mean of 3.7 minutes and a standard deviation of 1.4 minutes. Ninety-five percent of all customers entering the bank speak to a teller within how many minutes?

Calculator Solution:

- (S1) Let X be a normal random variable with $\mu = 3.7$ and $\sigma = 1.4$. Find the value of a such that $P(X < a) = .95$.
(S2) Use the function `invNorm` (in the `DISTR`; `DISTR` menu). Figure 6.16 shows that 95% of all customers entering the bank will be waited on within approximately 6 minutes.

Note: Figure 6.17 illustrates the solution to this problem using `ShadeNorm`. $\diamond \square$

The z -score is used throughout statistics in a variety of ways. The main text uses $z(\alpha)$ to represent a specific z -score, or point on the measurement axis for a standard normal random variable, such that there is α of the area, or probability, to the right of $z(\alpha)$. Therefore, there is $1 - \alpha$ of the area to the left of $z(\alpha)$. The TI-83 Plus function `invNorm` (inverse

cumulative probability for a normal random variable) may be used to find a specific z -score.

Example: Find the numerical values of $z(0.05)$ and $z(0.60)$.

Calculator Solution:

$$(S1) P(Z > z(0.05)) = 0.05 \implies P(Z \leq z(0.05)) = 1 - 0.05 = 0.95.$$

$$z(0.05) = 1.6449 \text{ (Figure 6.18).}$$

$$(S2) P(Z > z(0.60)) = 0.60 \implies P(Z \leq z(0.60)) = 1 - 0.60 = 0.40.$$

$$z(0.60) = -.2533 \text{ (Figure 6.19).} \quad \square$$

In certain cases a normal distribution may be used to approximate binomial probabilities. The number of trials should be large and as a general rule of thumb both np and $n(1 - p)$ should be greater than or equal to 5. In addition, a continuity correction should be applied when using a continuous (normal) random variable to estimate a discrete (binomial) random variable.

Example: A drug manufacturer claims that only 5% of the patients using a particular drug will experience adverse reactions. Doctors at a large university hospital decide to use the drug on 250 randomly selected patients. Use the normal approximation to the binomial distribution to find the probability that 15 or fewer of the 250 patients experience adverse reactions.

Calculator Solution:

(S1) Let X be the number of patients who experience adverse reactions. Assume X is a binomial random variable with $n = 250$ and $p = .05$. The number of trials is large and $np = (250)(.05) = 12.5 \geq 5$; $n(1 - p) = (250)(.95) = 237.5 \geq 5$.

$$(S2) \mu = np = (250)(.05) = 12.5$$

$$\sigma^2 = np(1 - p) = (250)(.05)(.95) = 11.875.$$

(S3) X is approximately distributed as a normal random variable with mean 12.5 and standard deviation $\sqrt{11.875}$.

$$(S4) P(X \leq 15) \approx P(X \leq 15.5) = P\left(\frac{X - 12.5}{\sqrt{11.875}} \leq \frac{15.5 - 12.5}{\sqrt{11.875}}\right) \\ = P(Z \leq .8706) = .8080 \text{ (Figure 6.20).}$$

Note:

(N1) This problem may be solved on the TI-83 Plus *without* the normal approximation by using the function `binomcdf` (Figure

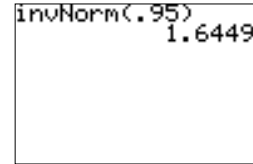


Figure 6.18: $z(0.05)$.

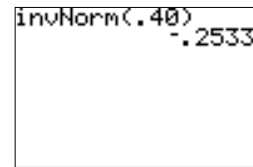


Figure 6.19: $z(0.60)$.

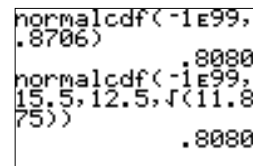


Figure 6.20: Solution using the normal approximation.

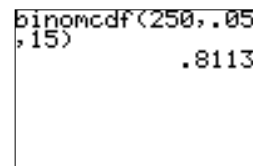


Figure 6.21: *Exact* probability using the binomial distribution.

6.21).

- (N2) On the TI-83 Plus, `binompdf` and `binomcdf` accept *very large* values of n , almost eliminating the need to approximate a binomial probability.
- (N3) Since $p = .05$ is *far away* from $.5$, the approximation is not as good as one might expect for such a large value of n . $\diamond \square$

Note: A normal distribution may also be used to approximate a Poisson random variable. The approximation may be used when the Poisson mean exceeds 3. The approximation improves as μ increases and the continuity correction should be applied. \diamond

Chapter 7

Sample Variability

A **random sample** of size n is chosen from a population so that every possible sample of fixed size n has an equal probability of being selected. Random sampling is one of the most widely used sampling techniques and is a basic assumption in many statistical procedures.

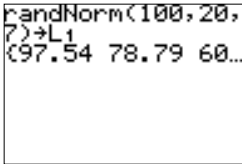
A table of random numbers may be used to help select a random sample. However, the TI-83 Plus has several built-in functions useful for generating random numbers and constructing a random sample. The function **rand** returns a random number between 0 and 1, and **randInt** returns a random integer between a lower and upper bound. **randBin** returns a random number drawn from a specified binomial distribution and **randNorm** generates a random number from a specified normal distribution. These four functions accept an optional argument for *number of trials* or *number of simulations*, and they are found in the **MATH**; PRB menu.

Example: Use the calculator to draw a random sample of size 7 from a normal probability distribution with a mean $\mu = 100$ and standard deviation $\sigma = 20$.

Calculator Solution:

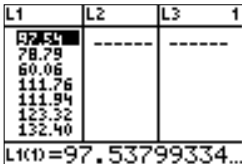
- (S1) On the Home Screen, select **randNorm** (in the **MATH**; PRB menu) and enter the mean, standard deviation, and sample size. Store the resulting sample in list L1 (Figure 7.1).
- (S2) The random sample is displayed on the Home Screen and may also be viewed in the stat list editor (Figure 7.2). □

The calculator program **RANDOM** may be used to generate a random sample of size n ($n \leq 999$) from a set of integers between a lower bound and an upper bound. The program also allows the option of sampling with replacement.



```
randNorm(100,20,
7)→L1
{97.54 78.79 60.06
111.76 111.94
123.32 132.40}
```

Figure 7.1: Random sample from a normal distribution.



L1	L2	L3	1
97.53799334	-----	-----	
78.79			
60.06			
111.76			
111.94			
123.32			
132.40			

Figure 7.2: Random sample viewed in the stat list editor.

```

LOW BND: 1
UPP BND: 6
SAMPLE SIZE: 5

```

Figure 7.3: Lower bound, upper bound, and sample size.

```

SAMPLE
1:W/OOUT REPLACE
2:WITH REPLACE

```

Figure 7.4: Sample without or with replacement.

L1	L2	L3	1
5.00	-----	-----	
2.00			
2.00			
6.00			
6.00			

L1()=5			

Figure 7.5: Five rolls in the stat list editor.

```

LOW BND: 1
UPP BND: 524
SAMPLE SIZE: 6

```

Figure 7.6: Lower bound, upper bound, and sample size.

```

SAMPLE
1:W/OOUT REPLACE
2:WITH REPLACE

```

Figure 7.7: Sample without replacement.

L1	L2	L3	1
23.00	-----	-----	
60.00			
365.00			
517.00			
332.00			
164.00			

L1()=23			

Figure 7.8: The 6 voter numbers.

Example: Use the program **RANDOM** to simulate rolling five dice.

Calculator Solution:

- (S1) Execute the program **RANDOM**. Input the lower bound, upper bound, and sample size (Figure 7.3).
- (S2) Select the type of sample: with or without replacement. In this example, *each* die may result in a 1–6, so the sampling is done *with* replacement (Figure 7.4).
- (S3) The 5 rolls are stored in the built-in list **L1**. The numbers may be viewed on the Home Screen or in the stat list editor (Figure 7.5). The resulting rolls are one 5, two 2's, and two 6's. □

Example: Six voters are to be selected at random from a list of 524 registered voters in a small town. The voters will be asked their opinion on a proposed new stop light in the center of town. Select a random sample of 6 voters.

Calculator Solution:

- (S1) Assume each voter in the town has been assigned a number from 1 through 524. Execute the program **RANDOM**.
- (S2) Input the lower bound, upper bound, and sample size (Figure 7.6).
- (S3) Select the type of sample: with or without replacement. In this problem, each voter may be selected only once, so the sampling is done without replacement (Figure 7.7).
- (S4) The random sample of size 6 is stored in the list **L1** and may be viewed on the Home Screen or in the stat list editor (Figure 7.8). Voters 23, 60, 365, 517, 332, and 164 should be selected and asked about the new stop light. □

In order to use the numbers in a sample to estimate a parameter that describes a population, the sample should be representative of the population. Even with a random sample, it is very unlikely that a sample statistic would exactly equal the population characteristic, or parameter, being estimated. The properties of a sample statistic are important in order to judge how *close* the observed value is to the population characteristic. The **sampling distribution of a sample statistic** is the

distribution of values for a sample statistic obtained from repeated samples, all of the same size and all drawn from the same population.

Suppose the sample mean, \bar{x} , is used to estimate a population mean, μ . The sample mean is subject to variability, that is, \bar{x} will vary from sample to sample. \bar{x} is really a specific value of a random variable. In some cases the exact distribution of this random variable, the sample mean, may be obtained. To investigate the **sampling distribution of sample means** one might consider several sample means, construct a histogram, compute the mean, variance, and standard deviation of the means, and compare these values with the population parameters.

Example: Consider a population consisting of five equally likely integers: 1, 2, 3, 4, and 5. The population mean is $\mu = 3.00$, variance $\sigma^2 = 2.00$, and standard deviation $\sigma = 1.41$. The table below lists 30 sample means, each from a random sample of size 5, with replacement.

Sample Means									
3.8	2.2	2.8	2.4	2.8	3.4	3.4	3.0	3.8	2.2
2.2	2.8	3.0	2.8	2.6	4.4	2.0	2.0	2.8	3.0
3.4	3.8	3.0	3.6	3.2	3.2	3.8	2.4	2.0	3.6

Construct a histogram of the means, find the mean, variance, and standard deviation of the means and compare these statistics to the population parameters.

Calculator Solution:

- (S1) Enter the sample means in the list L1. Use **1-Var Stats** to compute the summary statistics for this data (Figure 7.9).
- (S2) The sample mean of the means is 2.98, the sample standard deviation is .6376, and the sample variance is $(.6376)^2 = .4065$.
- (S3) In the **Plot1** menu select **On**, **Type** histogram, **Xlist:L1**, and **Freq:1** (Figure 7.10).
- (S4) Select appropriate **WINDOW** settings (Figure 7.11) and press **GRAPH** to display the histogram (Figure 7.12).
- (S5) The sample mean of the means is very close to the population mean. The distribution of the sample means is much more compact than the original distribution. \square

The **Central Limit Theorem** is one of the most important ideas in statistics. Let \bar{X} be the sample mean for a sample of size n from a population with mean μ and standard deviation σ .

- (C1) The mean of \bar{X} is μ : $\mu_{\bar{X}} = \mu$.

Figure 7.9: 1-Var Stats for sample means.

Figure 7.10: Plot1 settings.

Figure 7.11: WINDOW settings.

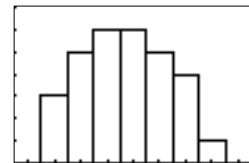


Figure 7.12: Histogram of sample means.

- (C2) The standard deviation of \bar{X} is given by: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$.
- (C3) If the sampled population is normal, then the sampling distribution of \bar{X} is normal for any sample size n .
- (C4) The Central Limit Theorem states: if n is sufficiently large, the sampling distribution of \bar{X} is *approximately* normal regardless of the sampled population.

Note:

- (N1) The standard deviation of \bar{X} , a measure of variability of the sampling distribution of sample means, is called the **standard error of the mean**: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$.
- (N2) In most real-world applications, the population standard deviation is unknown. An estimate of the standard error of the mean is: $s_{\bar{X}} = s/\sqrt{n}$.

Example: The height of kindergarten children has a distribution with mean 39 inches and a standard deviation of 3 inches. A random sample of 36 kindergarten students is selected and their heights are recorded. What is the approximate probability that the sample mean for the 36 students will be:

- (E1) greater than 39.75 inches.
 (E2) less than 37.5 inches.

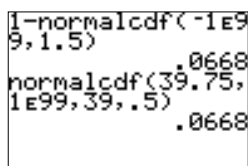


Figure 7.13:
 $P(\bar{X} > 39.75)$.

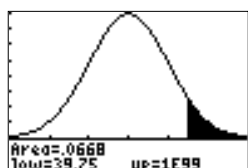


Figure 7.14: ShadeNorm used to illustrate $P(\bar{X} > 39.75)$.

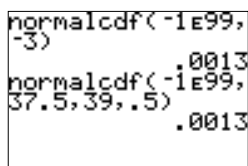


Figure 7.15:
 $P(\bar{X} < 37.5)$.

Calculator Solution:

- (S1) $\mu_{\bar{X}} = 39$ and $\sigma_{\bar{X}} = \frac{3}{\sqrt{36}} = .5$.
- (S2) Since n is large, the distribution of \bar{X} is approximately normal.
- (S3) $P(\bar{X} > 39.75) = P\left(\frac{\bar{X} - 39}{.5} > \frac{39.75 - 39}{.5}\right) = P(Z > 1.5)$
 $= 1 - P(Z \leq 1.5) = .0668$ (Figure 7.13).

Note: Figure 7.14 illustrates this probability with ShadeNorm. \diamond

- (S4) $P(\bar{X} < 37.5) = P\left(\frac{\bar{X} - 39}{.5} < \frac{37.5 - 39}{.5}\right)$
 $= P(Z < -3.00) = .0013$ (Figure 7.15).

Note: If the actual sample mean were 37.5 inches, this small probability suggests the claim of $\mu = 39$ inches is false. $\diamond \square$

Chapter 8

Introduction to Statistical Inferences

A **point estimate for a parameter** is the value of the corresponding sample statistic. Although a point estimate serves as a best guess, it does not convey a sense of reliability, or confidence, or how close the estimate is to the true population parameter. An **interval estimate** is a range of numbers used to estimate the true value of a population parameter. The values that bound this interval are statistics calculated from the sample used as the basis for the estimation. The **level of confidence** ($1 - \alpha$) is the probability that the sample to be selected yields an interval that includes the parameter being estimated. A **confidence interval** is an interval estimate with a specified level of confidence.

If the sampled population is normal or the sample size is large, the formula below may be used to construct a confidence interval for a population mean. Let \bar{X} be the sample mean for a sample of size n drawn from a population with mean μ and standard deviation σ . A large sample $100(1 - \alpha)\%$ confidence interval for μ has endpoints

$$\bar{x} \pm z(\alpha/2) \cdot \frac{\sigma}{\sqrt{n}}.$$

Note:

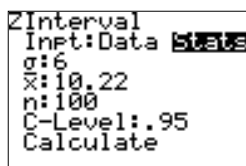
- (N1) $z(\alpha/2)$ is the **confidence coefficient**, the number of multiples of the standard error needed to formulate an interval estimate of the correct width to have a level of confidence of $1 - \alpha$. It is the point on the measurement axis in a standard normal distribution so that $P(Z > z(\alpha/2)) = \alpha/2$.
- (N2) $z(\alpha/2) \cdot (\sigma/\sqrt{n})$ is one-half the width of the confidence interval, the **maximum error of estimate** E .

- (N3) $\bar{x} - z(\alpha/2) \cdot (\sigma/\sqrt{n})$: **lower confidence limit** (LCL),
 $\bar{x} + z(\alpha/2) \cdot (\sigma/\sqrt{n})$: **upper confidence limit** (UCL).

The endpoints of the confidence interval in the formula above may be computed directly on the calculator Home Screen. Compute \bar{x} and check a table of common confidence coefficients for $z(\alpha/2)$, and evaluate the appropriate arithmetic expression. However, the built-in calculator function `ZInterval` (in the `[STAT]`; TESTS menu) is more general and speeds up the process for finding this confidence interval. Most of the TI-83 Plus confidence interval functions work with summary statistics or a list containing data.

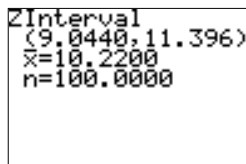
Example: The student activities office at a local state university would like to estimate the mean distance traveled to school by commuters. A random sample of 100 commuting students was selected and the one-way distance each commuted was obtained. The resulting sample mean distance was 10.22 miles. Find a 95% confidence interval for the mean one-way distance traveled by commuter students at the school. Use $\sigma = 6$ miles.

Calculator Solution:



```
ZInterval
Inpt:Data [STAT]
sigma:6
x:10.22
n:100
C-Level:.95
Calculate
```

Figure 8.1: ZInterval input screen.



```
ZInterval
(9.0440, 11.396)
x=10.2200
n=100.0000
```

Figure 8.2: Confidence interval for μ .

- (S1) The population parameter of interest is the mean one-way distance traveled by all commuting students. The distribution of the sampled population is unknown, but the sample size is large ($n = 100 \geq 30$). The confidence interval is based on the standard normal random variable.
- (S2) Select `[STAT]`; TESTS; ZInterval. On the ZInterval input screen highlight `Stats` (since only the summary statistics are known). Enter the values of σ , \bar{x} , n , and the confidence level (`C-Level`) (Figure 8.1).
- (S3) Position the blinking cursor on `Calculate` and press `[ENTER]`. The interval estimate, \bar{x} , and n are displayed on the Home Screen (Figure 8.2).
- (S4) (9.044, 11.396) is a 95% confidence interval for the true mean one-way distance traveled by commuting students. \square

Example: A certain adjustment to a machine will change the length of the parts produced but will not affect the standard deviation. The length of the parts is normally distributed and the standard deviation is 0.5 mm. After an adjustment is made, a random sample is taken to determine the mean length of parts now being produced. The resulting lengths are:

75.0 76.0 75.0 77.0 75.4 76.3 77.0 74.9 76.5 75.8

Find a 99% confidence interval for the mean length of parts produced.

Calculator Solution:

- (S1) The population parameter of interest is the mean length of parts produced and the distribution of the sampled population is normal. The confidence interval is based on the standard normal random variable.
- (S2) Enter the data in the list PARTS (Figure 8.3).
- (S3) On the ZInterval input screen highlight **Data** and use $\sigma = 0.5$. Enter the list containing the data, the frequency of each observation, and the confidence level (Figure 8.4).
- (S4) Highlight **Calculate** and press **ENTER**. The interval estimate, \bar{x} , s , and n are displayed on the Home Screen (Figure 8.5).
- (S5) (75.483, 76.297) is a 99% confidence interval for the mean length of parts produced. \square

PARTS	----	----	1
75.0			
76.0			
75.0			
77.0			
75.4			
76.3			
77.0			
76.000			
76.500			
75.800			
PARTS(O) =			74.9

Figure 8.3: PARTS data.

ZInterval
Inpt: DATA Stats
σ : .5
List: PARTS
Freq: 1
C-Level: .99
Calculate

Figure 8.4: ZInterval input screen.

ZInterval
(75.483, 76.297)
\bar{x} = 75.8900
S_x = .8048
n = 10.0000

Figure 8.5: Confidence interval for μ .

A **hypothesis test** is a formal process used to test the validity of a statement, or claim, about a population parameter. Using information in a sample, the claim is either rejected or not rejected.

There are four parts to every hypothesis test:

- (P1) The **null hypothesis**, H_0 , is a statement that a population parameter has a specific value, assumed to be true.
- (P2) The **alternative hypothesis**, H_a , is a statement about the same population parameter, usually the complement of H_0 , all other values of the population parameter.
- (P3) The **test statistic**, TS, is a random variable calculated from the sample data, used as a basis for deciding whether or not to reject the null hypothesis.
- (P4) The **critical region** (CR), or **rejection region** (RR), is a set or interval of numbers selected in such a way that if the value of the test statistic lies in the critical region we reject the null hypothesis.

If the sampled population is normal or the sample size is large and σ is known, the following procedure may be used to test a claim about a population mean μ :

$$H_0: \mu = \mu_0$$

$$H_a: \mu > \mu_0, \quad \mu < \mu_0, \quad \mu \neq \mu_0$$

$$\text{TS: } z^* = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

Note:

- (N1) When conducting this hypothesis test, select only one of the three possible alternative hypotheses. There are two one-tailed tests and one two-tailed test to choose from and three corresponding critical regions.
- (N2) α is the **significance level** of the test, the probability of committing a type I error. The test statistic is **significant** if it lies in the critical region.
- (N3) $z(\alpha)$ is a **critical value**, a boundary value of the critical region.
- (N4) The **p-value** is the probability that the test statistic could be the value it is or a more extreme value (in the direction of the alternative hypothesis) when the null hypothesis is true. \diamond

The value of the test statistic, z^* , may be computed on the Home Screen by evaluating an appropriate arithmetic expression. However, the TI-83 Plus built-in hypothesis test procedures, in the **[STAT]**; TESTS menu, are more efficient and will be used to solve hypothesis test examples that follow.

Example: The admissions office at a local hospital claims the mean age of its patients is 42 years. In an attempt to check this claim, an insurance provider selects a random sample of 120 patients and records their ages. The sample mean is 44.2. Is there sufficient evidence to conclude the mean age of patients at the hospital is larger than 42? Use $\alpha = .05$ and $\sigma = 20$.

Calculator Solution:

- (S1) This is a one-sided, or one-tailed, test about a population mean. The sampled distribution is not known but the sample size is large. The test statistic is z^* .
- (S2) The four parts of the hypothesis test are:

$$H_0: \mu = 42 (\leq)$$

$$H_a: \mu > 42$$

$$\text{TS: } z^* = \frac{\bar{X} - 42}{\sigma/\sqrt{n}}$$

$$\text{CR: } z^* \geq z(\alpha) = z(0.05) = 1.645$$

- (S3) In the **[STAT]**; TESTS menu, select Z-Test. The TI-83 Plus Z-Test input is either data in a list (**Data**) or summary statistics (**Stats**). The summary statistics (and σ) are known in this example. Highlight **Stats** and enter values for μ_0 , σ , \bar{x} , and n . Highlight the one-sided upper-tailed alternative hypothesis, position the blinking cursor on **Calculate**, and press **[ENTER]** (Figure 8.6).

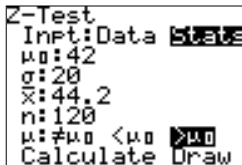


Figure 8.6: Z-Test input screen.

- (S4) The hypothesis test results are summarized on the Home Screen (Figure 8.7).
- (S5) Decision: do not reject H_0 .
Conclusion: The value of the test statistic, $z^* = 1.205$, does not lie in the critical region. There is no evidence to suggest the mean age of patients at the hospital is greater than 42.

Note:

- (N1) The calculator hypothesis test results screen also contains the p -value, or observed significance level, for the test. The p -value is also thought of as the smallest value α (smallest level of significance) for which the null hypothesis would be rejected. Here, $P = .1141 > .05$, which indicates there is no evidence in favor of the alternative hypothesis.
- (N2) From the Z-Test input screen, you may also **Draw** the results of the hypothesis test. The TI-83 Plus will automatically set the WINDOW, **Draw** a standard normal probability function, compute and display the value of the test statistic z^* and the p -value, and shade the area under the curve corresponding to the p -value (Figure 8.8). ◇ □

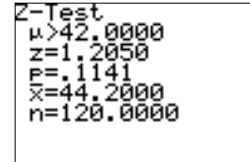


Figure 8.7: Hypothesis test results.

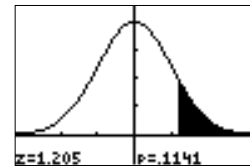


Figure 8.8: Hypothesis test Draw results.

Example: A manufacturer claims the weight of a full box of its cereal is normally distributed with mean 10 ounces and standard deviation 0.27 ounces. A random sample of 18 boxes is selected and the weight of each box is given below.

Weights					
9.71	10.07	9.20	9.81	10.29	10.01
9.93	9.78	10.10	9.40	9.63	9.66
10.02	9.88	10.00	9.95	9.91	9.73

Is there any evidence to suggest the mean weight of the cereal boxes is less than 10 ounces? Use a 0.01 level of significance.

Calculator Solution:

- (S1) This is a one-tailed test about a population mean. The sampled distribution is normal and the standard deviation is known. The test statistic is z^* .
- (S2) The four parts of the hypothesis test are:

$$H_0: \mu = 10 (\geq)$$

$$H_a: \mu < 10$$

$$\text{TS: } z^* = \frac{\bar{X} - 10}{\sigma/\sqrt{n}}$$

```

Z-Test
Inpt:DATA Stats
μ₀:10
σ: .27
List:BOXES
Freq:1
μ:≠μ₀ <μ₀ >μ₀
Calculate Draw

```

Figure 8.9: Z-Test input screen.

```

Z-Test
μ<10.0000
z=-2.5491
P=.0054
x̄=9.8378
Sx=.2601
n=18.0000

```

Figure 8.10: Hypothesis test results.

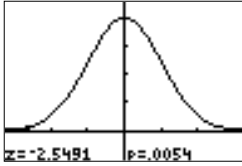


Figure 8.11: Hypothesis test Draw results.

$$\text{CR: } z^* \leq -z(\alpha) = -z(0.01) = -2.33$$

- (S2) Enter the data in the list BOXES.
- (S3) Select **STAT**; TESTS; Z-Test. On the Z-Test input screen highlight Data, set $\mu_0 = 10$, enter the standard deviation, use the list BOXES, set the frequency to 1, and select the one-sided lower-tailed alternative hypothesis (Figure 8.9).
- (S4) Calculate the Z test. The hypothesis test results are displayed on the Home Screen (Figure 8.10).
- (S5) Decision: Reject H_0 .

Conclusion: The value of the test statistic, $z^* = -2.5491$, lies in the critical region (equivalently, $P = .0054 < .01$). There is evidence to suggest the mean weight of the cereal boxes is less than 10 ounces.

Note: The Draw hypothesis test results are shown in Figure 8.11.

◇ □

Chapter 9

Inferences Involving One Population

Inferences about a population mean μ are based on the sample mean \bar{x} and information obtained from the sampling distribution of sample means. If the population standard deviation, σ , is known, and the sampled population is normal or the sample size is sufficiently large, then inferences are based on the standard normal distribution. The test statistic is z^* .

If the population standard deviation, σ , is unknown, the standard error σ/\sqrt{n} is also unknown. In this case the sample standard deviation, s is used as a point estimate for σ , and s/\sqrt{n} is used as an estimate for the standard error of the mean. Inferences about a population mean are based on the **Student's t statistic**.

Suppose a random sample of size n is selected from a normal population. If the population standard deviation is unknown, a $100(1-\alpha)\%$ confidence interval for μ has endpoints

$$\bar{x} \pm t(n-1, \alpha/2) \cdot \frac{s}{\sqrt{n}}.$$

Note:

- (N1) $t(n-1, \alpha/2)$ is the critical value in the t distribution with $n-1$ degrees of freedom.
- (N2) This confidence interval may be constructed using the TI-83 Plus calculator function **TInterval**, in the **[STAT]; TESTS** menu.
- (N3) As the sample size, n , increases, $t(n-1, \alpha/2) \rightarrow z(\alpha/2)$. \diamond

Example: Gasoline pumped from a supplier's pipeline is supposed to have a specified octane rating. On fourteen randomly selected days, a sample of

gasoline was analyzed and the octane rating recorded. The data is given below.

Octane Ratings						
88.6	86.4	87.2	88.4	87.2	87.6	86.8
86.1	87.4	87.3	86.4	86.6	87.1	88.3

Suppose the octane ratings have a normal distribution. Find a 98% confidence interval for the true mean octane rating of gasoline from the supplier's pipeline.

```

TIInterval
Inpt:Stats
List:OCTAN
Freq:1
C-Level:.98
Calculate
  
```

Figure 9.1: TIInterval input screen.

```

TIInterval
(86.694, 87.792)
x-bar=87.2429
Sx=.7753
n=14.0000
  
```

Figure 9.2: Confidence interval for μ .

```

TIInterval
Inpt:Data Stats
x-bar:14.5
Sx:3.1
n:21
C-Level:.95
Calculate
  
```

Figure 9.3: TIInterval input screen if only summary statistics are known.

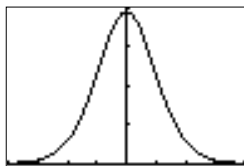


Figure 9.4: A t probability function.

```

tcdf(-.5, 1.2, 17)
.5650
  
```

Figure 9.5: Use of `tcdf` to compute probability.

Calculator Solution:

- (S1) The population parameter of interest is the mean octane rating of gasoline from the supplier's pipeline. The distribution of the sampled population is assumed normal and σ is unknown. The confidence interval is based on the t distribution.
- (S2) Enter the data in the list OCTAN.
- (S3) Select `STAT`; TESTS; TIInterval. On the TIInterval input screen highlight Data, select the list OCTAN, set the frequency to 1, and enter the confidence level (Figure 9.1).
- (S4) Position the blinking cursor on Calculate and press `ENTER`. The interval estimate, \bar{x} , s , and n are displayed on the Home Screen (Figure 9.2).
- (S5) (86.694, 87.792) is a 98% confidence interval for the true mean octane rating of gasoline from the supplier's pipeline. \square

Note:

- (N1) The function `TIInterval` may also be used to construct a confidence interval if only summary statistics are known (and the sampled distribution is normal). Figure 9.3 shows a typical TIInterval input screen for `Inpt:Stats`.
- (N2) In the `DISTR`; `DISTR` menu the function `tpdf` returns a value of the probability distribution function for a t distribution with specified (ν) degrees of freedom; function with arguments: `tpdf(x, ν)`. Figure 9.4 shows the graph of the probability distribution function for a t random variable with 11 degrees of freedom (WINDOW settings: $[-4, 4] \times [0, .4]$).
- (N3) The function `tcdf` computes the probability of selecting a value between a lower (a) and upper (b) bound for a t distribution with specified (ν) degrees of freedom; function with arguments: `tcdf(a, b, ν)`. Figure 9.5 shows the probability of selecting a value between $-.5$ and 1.2 from a t distribution with 17 degrees of freedom is $.5650$.
- (N4) In the `DISTR`; `DRAW` menu the function `Shade_t` draws the probabil-

ity function for a t distribution with specified (ν) degrees of freedom, shades the area between a lower (a) and upper (b) bound, and computes the shaded area; function and arguments: `Shade_t(a,b, ν)`. Figure 9.6 illustrates the use of `Shade_t` to visualize the probability found in note (N3).

- (N5) As the degrees of freedom increase, the t distribution approaches a standard normal distribution. Figure 9.7 illustrates this concept. The top, dotted graph is of the standard normal distribution function. The remaining two graphs are the probability functions for a t distribution with 2 and 7 degrees of freedom. \diamond

Suppose a random sample of size n is selected from a normal population. If the population standard deviation is unknown, the t statistic is used to complete a hypothesis test about a population mean. The calculator function `T-Test` (in the `STAT`; `TESTS` menu) may be used with data stored in a list or with summary statistics.

The hypothesis test procedure:

$$H_0: \mu = \mu_0$$

$$H_a: \mu > \mu_0, \quad \mu < \mu_0, \quad \mu \neq \mu_0$$

$$\text{TS: } t^* = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

$$\text{CR: } t^* \geq t(n-1, \alpha), \quad t^* \leq -t(n-1, \alpha), \quad |t^*| \geq t(n-1, \alpha/2)$$

Example: Homes in a nearby college town have a mean value of \$88,950. It is assumed that homes closer to the college have a higher value. To test this theory, a random sample of 12 homes is selected from the college area. Their mean valuation is \$92,460 with standard deviation \$5,200. Is there any evidence to suggest the mean value of homes in the college area is higher than the nearby town? Complete the hypothesis test using $\alpha = .05$ and assume prices are normally distributed.

Calculator Solution:

- (S1) This is a one-sided upper-tailed test about a population mean. The sampled distribution is assumed normal and σ is unknown. The test statistic is t^* .

- (S2) The four parts of the hypothesis test are:

$$H_0: \mu = 88950 (\leq)$$

$$H_a: \mu > 88950$$

$$\text{TS: } t^* = \frac{\bar{X} - 88950}{S/\sqrt{n}}$$

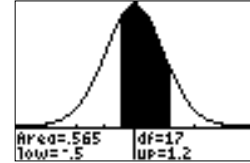


Figure 9.6: Use of `Shade_t` to illustrate (and compute) probability.

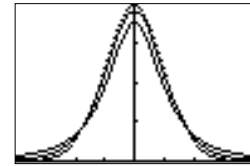


Figure 9.7: The t distribution approaches a standard normal distribution.

```

T-Test
Inpt:Data Stats
μ₀:88950
x̄:92460
Sx:5200
n:12
μ:≠μ₀ <μ₀ >μ₀
Calculate Draw

```

Figure 9.8: T-Test input screen.

```

T-Test
μ>88950.0000
t=2.3383
P=.0196
x̄=92460.0000
Sx=5200.0000
n=12.0000

```

Figure 9.9: Hypothesis test results.

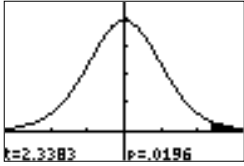


Figure 9.10: Hypothesis test Draw results.

```

1-PropZInt
x:45
n:200
C-Level:.99
Calculate

```

Figure 9.11: 1-PropZInt input screen.

```

1-PropZInt
(.1489,.3011)
p̂=.2250
n=200.0000

```

Figure 9.12: Confidence interval for p .

$$\text{CR: } t^* \geq t(n-1, \alpha) = t(11, .05) = 1.80$$

(S3) In the **[STAT]**; TESTS menu, select T-Test. Since summary statistics are known, highlight Stats. Enter values for μ_0 , \bar{x} , S_x (sample standard deviation), and n . Select the one-sided upper-tailed alternative hypothesis and Calculate the results (Figure 9.8).

(S4) The hypothesis test results are shown in Figure 9.9.

(S5) Decision: reject H_0 .

Conclusion: The value of the test statistic, $t^* = 2.3383$, lies in the critical region (equivalently, $P = .0196 < .05$). There is evidence to suggest the mean valuation of homes in the vicinity of the college is higher than \$88,950.

Note: The Draw results are shown in Figure 9.10. $\diamond \square$

The observed (sample) binomial probability, or sample proportion, p' , may be used as a point estimate for a binomial parameter p , $P(\text{success})$. If the sample size is greater than 20 but less than 10% of the population, and the products np' and $n(1-p')$ are greater than 5, an interval estimate may be constructed using the standard normal distribution.

A $100(1-\alpha)\%$ confidence interval for a binomial parameter p has endpoints

$$p' \pm z(\alpha/2) \cdot \sqrt{\frac{p'q'}{n}}$$

where $z(\alpha/2)$ is the critical value from a standard normal distribution, p' is the sample binomial probability, and $q' = 1 - p'$. The calculator function 1-PropZInt (in the **[STAT]**; TESTS menu) may be used to compute this interval estimate for p .

Example: In a sample of 200 randomly selected residents, only 45 favored a proposed Good Samaritan law. Construct a 99% confidence interval for the true proportion of all residents who support the proposed new law. Is it reasonable to conclude that at least 35% of the residents support the proposed law?

Calculator Solution:

(S1) The parameter of interest is the proportion of residents who support the proposed law. The sample size is large; the confidence interval for p is appropriate.

(S2) In the 1-PropZInt input screen, enter values for x (the number of successes), n , and the confidence level (Figure 9.11).

(S3) Calculate the confidence interval. The interval estimate, sample binomial probability, \hat{p} , and number of trials are displayed on the Home Screen (Figure 9.12).

- (S4) (.1489, .3011) is a 99% confidence interval for the true proportion of residents who favor the new law. .35 is *not* in the interval and is more than the entire interval of numbers. This suggests that less than 35% of all residents support the new law. \square

The sample binomial probability may also be used to test a hypothesis about a binomial parameter p . If the sample size is large and the sample binomial probability is approximately normal, a hypothesis test about p is based on the standard normal distribution. The built-in calculator procedure `1-PropZTest` produces `Calculate` and `Draw` results.

Hypothesis test procedure about a binomial parameter p :

$$H_0: p = p_0$$

$$H_a: p > p_0, \quad p < p_0, \quad p \neq p_0$$

$$\text{TS: } z^* = \frac{p' - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

Example: Many people sleep-in on the weekends to make up for *short nights* during the work week. The Better Sleep Council reports that 61% of us get more than seven hours of sleep per night on the weekend. A random sample of 350 adults found that 235 had more than seven hours each night last weekend. At the .05 level of significance, does this evidence show that more than 61% get seven or more hours per night on the weekend?

Calculator Solution:

- (S1) This is a one-sided upper-tailed test about a binomial parameter. The sample size is large and the sample binomial probability is assumed normal. The test statistic is z^* .

- (S2) The four parts of the hypothesis test are:

$$H_0: p = .61 (\leq)$$

$$H_a: p > .61$$

$$\text{TS: } z^* = \frac{p' - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$$\text{CR: } z^* \geq z(\alpha) = z(.05) = 1.645$$

- (S3) Select `[STAT]`; `TESTS`; `1-PropZTest`. Enter values for p_0 , x (the number of successes), and n . Select the one-sided upper-tailed al-

```

1-PropZTest
P0: .61
x: 235
n: 350
PROP≠P0 <P0 >P0
Calculate Draw

```

Figure 9.13:
1-PropZTest input
screen.

```

1-PropZTest
PROP>.6100
z=2.3562
P=.0092
p̂=.6714
n=350.0000

```

Figure 9.14: Hypothesis
test results.

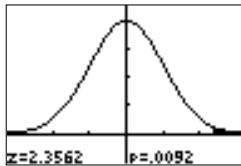


Figure 9.15: Hypothesis
test Draw results.

```

INPUT?
1: DATA
2: STATS

```

Figure 9.16: Input is
summary statistics.

```

S2 = 2.75
N = 30
C-LEVEL = .95

```

Figure 9.17: Sample
variance, sample size,
and confidence level.

```

CI ENDPOINTS
1.7442
4.9698
Done

```

Figure 9.18: Confidence
interval for σ^2 .

ternative hypothesis and **Calculate** the results (Figure 9.13).

(S4) The hypothesis test results are displayed on the Home Screen (Figure 9.14). \hat{p} is the sample binomial probability.

(S5) Decision: reject H_0 .

Conclusion: The value of the test statistic, $z^* = 2.3562$, lies in the critical region (equivalently, $P = .0092 < .05$). There is evidence to suggest that more than 61% of us get seven or more hours of sleep per night on the weekend.

Note: The **Draw** results are shown in Figure 9.15. $\diamond \square$

The sample variance s^2 may be used as a point estimate for a population variance σ^2 . If a sample of size n is drawn from a normal population, an interval estimate may be constructed based on the chi-square distribution. A $100(1 - \alpha)\%$ confidence interval for a population variance σ^2 is given by

$$\frac{(n-1)s^2}{\chi^2(n-1, \alpha/2)} < \sigma^2 < \frac{(n-1)s^2}{\chi^2(n-1, 1-\alpha/2)}$$

$\chi^2(n-1, \alpha/2)$ and $\chi^2(n-1, 1-\alpha/2)$ are critical values from a chi-square distribution with $n-1$ degrees of freedom. The calculator program **CHISQINT** may be used to compute this interval estimate for σ^2 . It will find the critical values and construct a confidence interval from summary statistics or from data stored in a calculator list.

Example: A commercial farmer harvests his entire field of vegetables at one time. Therefore, he would like to plant a variety of green beans that mature all at one time (small variance between maturity times of individual plants). A seed company has developed a new hybrid strain of green beans that it believes to be better for the commercial farmer. A random sample of 30 plants of the new hybrid showed a sample variance of 2.75 days. Find a 95% confidence interval for the variance of maturity times. Assume that maturity time is normally distributed.

Calculator Solution:

(S1) The population parameter of interest is the variance of maturity times for green bean plants. The distribution of the sampled population is assumed normal. The confidence interval is based on the χ^2 distribution.

(S2) Execute the program **CHISQINT**. At the **INPUT?** prompt, select the **STATS** option (Figure 9.16). Enter the sample variance, sample size, and confidence level (Figure 9.17).

(S3) The endpoints for the confidence interval are displayed on the Home Screen (Figure 9.18). (1.7442, 4.9698) is a 95% confidence interval for the population variance of maturity times. \square

Note:

- (N1) In the **[DISTR]**; **DISTR** menu the function $\chi^2\text{pdf}$ returns a value of the probability density function for a chi-square distribution with specified (ν) degrees of freedom; function with arguments $\chi^2\text{pdf}(x, \nu)$. Figure 9.19 shows the graph of the probability density function for a chi-square random variable with 7 degrees of freedom (**WINDOW** settings: $[0, 20] \times [0, .13]$).
- (N2) The function $\chi^2\text{cdf}$ computes the probability of selecting a value between a lower (a) and upper bound (b) for a chi-square distribution with specified (ν) degrees of freedom; function with arguments $\chi^2\text{cdf}(a, b, \nu)$. Figure 9.20 shows the probability of selecting a value between 3.4 and 9.7 from a chi-square distribution with 14 degrees of freedom is .2144.
- (N3) In the **[DISTR]**; **DRAW** menu the function **Shade** χ^2 draws the probability density function for a chi-square distribution with specified (ν) degrees of freedom, shades the area between a lower (a) and upper (b) bound, and computes the shaded area; function and arguments: **Shade** $\chi^2(a, b, \nu)$. Figure 9.21 illustrates the use of **Shade** χ^2 to visualize the probability found in note (N2). \diamond

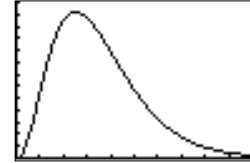


Figure 9.19: A chi-square probability density function.

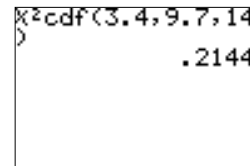


Figure 9.20: Use of $\chi^2\text{cdf}$ to compute probability.

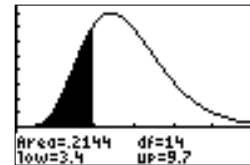


Figure 9.21: Use of **Shade** χ^2 to illustrate (and compute) probability.

If the sampled population is normal, a hypothesis test about a population variance is based on a chi-square distribution. The calculator program **CHISQTST** may be used to conduct this test using data stored in a list or with summary statistics.

Hypothesis test procedure about a population variance σ^2 :

$$H_0: \sigma^2 = \sigma_0^2$$

$$H_a: \sigma^2 > \sigma_0^2, \quad \sigma^2 < \sigma_0^2, \quad \sigma^2 \neq \sigma_0^2$$

$$\text{TS: } \chi^{2*} = \frac{(n-1)S^2}{\sigma_0^2}$$

$$\text{CR: } \chi^{2*} \geq \chi^2(n-1, \alpha), \quad \chi^{2*} \leq \chi^2(n-1, 1-\alpha),$$

$$\chi^{2*} \leq \chi^2(n-1, 1-\alpha/2) \quad \text{or} \quad \chi^{2*} \geq \chi^2(n-1, \alpha/2)$$

Example: An automobile manufacturer claims the miles per gallon for a specially designed high mileage, two-seat model has mean 40.5 with variance 12.3. A random sample of 15 such cars was obtained and tested. The observed miles per gallon are given in the table below.

Miles Per Gallon							
37.0	38.0	42.5	45.0	34.0	32.0	36.0	35.5
38.0	42.5	40.0	42.5	35.0	30.0	37.5	

```

INPUT?
1:DATA
2:STATS

```

Figure 9.22: Input from data in a list.

```

LIST: MILES
σ2 = 12.3

```

Figure 9.23: List name and σ_0^2 .

```

ALT HYP?
1:σ2 > σ2
2:σ2 < σ2
3:σ2 ≠ σ2

```

Figure 9.24: Alternative hypothesis.

```

H0: σ2 = 12.3000
HA: σ2 > 12.3000
TS = 20.2358
P = .1229

S2 = 17.7786
N = 15.0000

```

Figure 9.25: Hypothesis test results.

Is there any evidence to suggest the variance in miles per gallon is greater than 12.3? Complete the hypothesis test using $\alpha = .05$ and miles per gallon distribution is normal.

Calculator Solution:

(S1) This is a one-sided upper-tailed test about a population variance. The sampled distribution is assumed normal and the test statistic is χ^{2*} .

(S2) The four parts of the hypothesis test are:

$$H_0: \sigma^2 = 12.3 (\leq)$$

$$H_a: \sigma^2 > 12.3$$

$$\text{TS: } \chi^{2*} = \frac{(n-1)S^2}{12.3}$$

$$\text{CR: } \chi^{2*} \geq \chi^2(n-1, \alpha) = \chi^2(14, .05) = 23.7$$

(S3) Enter the data in the list MILES and execute the program CHISQTST. Select DATA at the INPUT? prompt (Figure 9.22).

(S4) Enter the name of the list containing the data and the value for σ_0^2 (Figure 9.23), and select the alternative hypothesis (Figure 9.24).

(S5) The hypothesis test results are displayed on the Home Screen (Figure 9.25).

(S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $\chi^{2*} = 20.2358$, does not lie in the critical region (equivalently, $P = .1229 > .05$). There is no evidence to suggest the variance in miles per gallon is greater than 12.3. \square

Chapter 10

Inferences Involving Two Populations

The TI-83 Plus has built-in hypothesis test procedures for comparing two population means (and for constructing confidence intervals for the difference between two means). The following notation will be used to describe these (and other) tests.

Parameter	Population 1	Population 2
Population mean	μ_1	μ_2
Population variance	σ_1^2	σ_2^2
Population standard deviation	σ_1	σ_2

Statistic	Sample 1	Sample 2
Sample size	n_1	n_2
Sample mean	\bar{x}_1	\bar{x}_2
Sample variance	s_1^2	s_2^2
Sample standard deviation	s_1	s_2

Suppose both populations are normally distributed and the data is *paired* (dependent samples). A $100(1 - \alpha)\%$ confidence interval for $\mu_d = \mu_1 - \mu_2$ has endpoints

$$\bar{d} \pm t(n - 1, \alpha/2) \cdot \frac{s_d}{\sqrt{n}}$$

where \bar{d} is the sample mean of the differences, s_d is the sample standard deviation of the differences, and n is the sample size (for both samples). This confidence interval is constructed by first computing the differences, then using the TI-83 Plus function (one-sample) **TInterval** (in the **STAT**; **TESTS** menu). This function accepts data stored in a list or summary statistics.

The hypothesis test about a mean difference:

$$H_0: \mu_d = \Delta_0$$

$$H_a: \mu_d > \Delta_0, \quad \mu_d < \Delta_0, \quad \mu_d \neq \Delta_0$$

$$\text{TS: } t^* = \frac{\bar{D} - \Delta_0}{S_d/\sqrt{n}}$$

$$\text{CR: } t^* \geq t(n-1, \alpha), \quad t^* \leq -t(n-1, \alpha), \quad |t^*| \geq t(n-1, \alpha/2)$$

The calculator function (one-sample) **T-Test** (in the **STAT**; **TESTS** menu) may be used to conduct this hypothesis test. Data or summary statistics may be used.

```

LBEFOR←LAFTER→LD
IFFS
{1.00 4.00 -2.0...

```

Figure 10.1: Calculation of differences.

BEFOR	AFTER	DIFFS	1
93.00	92.00	1.00	
106.00	102.00	4.00	
87.00	89.00	-2.00	
92.00	92.00	0.00	
102.00	101.00	1.00	
95.00	96.00	-1.00	
88.00	88.00	0.00	
Σ	105.00	5.00	
BEFOR(0) = 110			

Figure 10.2: Before, after, and difference in diastolic readings.

```

TInterval
Inpt:DATA Stats
List:DIFFS
Freq:1
C-Level:.98
Calculate

```

Figure 10.3: TInterval input screen.

```

TInterval
(-1.534, 3.5337)
x̄=1.0000
Sx=2.3905
n=8.0000

```

Figure 10.4: Confidence interval for μ_d .

Example: Salt-free diets are often prescribed for people with high blood pressure. The data in the table below was obtained from an experiment designed to estimate the reduction in diastolic blood pressure as a result of following a salt-free diet for two weeks. Assume diastolic readings are normally distributed.

Diastolic Readings								
Before	93	106	87	92	102	95	88	110
After	92	102	89	92	101	96	88	105

Find a 98% confidence interval for the mean reduction in diastolic blood pressure.

Calculator Solution:

- (S1) Diastolic readings are assumed normal and the samples are dependent. The confidence interval is based on the t distribution.
- (S2) Enter the data in the lists **BEFOR** and **AFTER**. Compute the differences on the Home Screen and store the results in the list **DIFFS** (Figures 10.1, 10.2).
- (S3) Select **STAT**; **TESTS**; **TInterval**. On the **TInterval** input screen highlight **Data**, select the list **DIFFS**, set the frequency to 1, and enter the confidence level (Figure 10.3).
- (S4) Position the blinking cursor on **Calculate** and press **ENTER**. The interval estimate, the sample mean difference (\bar{x}), the sample standard deviation for the differences (Sx), and n are displayed on the Home Screen (Figure 10.4).
- (S5) $(-1.534, 3.5337)$ is a 98% confidence interval for the mean difference in diastolic readings. \square

Example: To test the effect of a physical fitness course on one's physical ability, the number of sit-ups that a person could do in one minute, both

before and after the course, were recorded. Ten people were randomly selected and the difference data (before – after) is summarized by

$$n = 10, \quad \bar{d} = -7.00, \quad s_d = 5.7927.$$

Is there any evidence to suggest an improvement in physical ability (a greater number of sit-ups in one minute)? Use $\alpha = 0.01$.

Calculator Solution:

(S1) The number of sit-ups in one minute is assumed to be normally distributed and the samples are dependent. The test statistic is t^* .

(S2) The four parts of the hypothesis test are:

$$H_0: \mu_d = 0 (\geq)$$

$$H_a: \mu_d < 0$$

$$\text{TS: } t^* = \frac{\bar{D} - 0}{S_d/\sqrt{n}}$$

$$\text{CR: } t^* \leq -t(n-1, \alpha) = -t(9, 0.01) = -2.82$$

(S3) In the **[STAT]**; **TESTS** menu, select **T-Test**. Since summary statistics are known, highlight **Stats**. Enter values for μ_0 (μ_d), \bar{x} (\bar{d}), Sx (s_d), and n . Select the one-sided lower-tailed alternative hypothesis and **Calculate** the results (Figure 10.5).

(S4) The hypothesis test results are shown in Figure 10.6.

(S5) Decision: reject H_0 .

Conclusion: The value of the test statistic, $t^* = -3.8214$, lies in the critical region (equivalently, $P = .0020 < .01$). There is evidence to suggest the fitness course has improved physical ability.

Note: The **Draw** results are shown in Figure 10.7. $\diamond \square$

Suppose both populations are normally distributed, the samples are independent, and no assumptions are made about the population variances. A $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ has endpoints

$$(\bar{x}_1 - \bar{x}_2) \pm t(\text{df}, \alpha/2) \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where df is the approximate number of degrees of freedom,

$$\text{df} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}.$$

The calculator function **2-SampTInt** may be used to construct this confidence interval. The input may be data stored in lists or summary statistics.

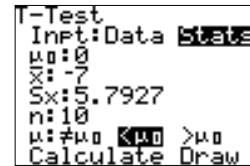


Figure 10.5: T-Test input screen.

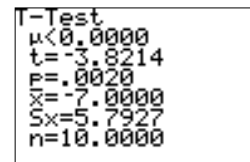


Figure 10.6: Hypothesis test results.

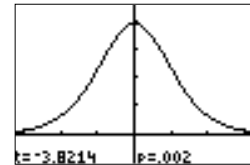


Figure 10.7: Hypothesis test Draw results.

The hypothesis test procedure:

$$H_0: \mu_1 - \mu_2 = \Delta_0$$

$$H_a: \mu_1 - \mu_2 > \Delta_0, \quad \mu_1 - \mu_2 < \Delta_0, \quad \mu_1 - \mu_2 \neq \Delta_0$$

$$\text{TS: } t^* = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$\text{CR: } t^* \geq t(\text{df}, \alpha), \quad t^* \leq -t(\text{df}, \alpha), \quad |t^*| \geq t(\text{df}, \alpha/2)$$

The calculator function `2-SampTTest` may be used to conduct this hypothesis test. The input may be data stored in lists or summary statistics.

Example: The heights (in inches) of 20 randomly selected women and 30 randomly selected men were independently obtained from the student body of a certain college in order to estimate the difference in their mean heights. The sample information is given in the following table.

Sample	Number	Mean	Standard Deviation
Female	20	63.8	2.18
Male	30	69.8	1.92

Assume the heights are approximately normally distributed for both populations. Find a 95% confidence interval for the difference between the mean heights.

Figure 10.8: 2-SampTInt input screen.

Figure 10.9: Confidence interval for $\mu_1 - \mu_2$.

Calculator Solution:

- (S1) Both populations are assumed to be approximately normal, and the samples were random and independently selected. No assumptions are made about the population variances. The confidence interval for the difference between the mean heights is based on the t distribution.
- (S2) Select `[STAT]`; `TESTS`; `2-SampTInt`. Highlight `Stats` and enter the summary statistics for each group. Enter the level of confidence and highlight `No` for pooled variance (no assumptions about the population variances). `Calculate` the confidence interval (Figure 10.8).
- (S3) The confidence interval and summary statistics are displayed on the Home Screen (Figure 10.9).
- (S4) $(-7.216, -4.784)$ is a 95% confidence interval for the difference between the mean heights (female – male). \square

Example: A study is designed to compare the academic success of college students who belong to fraternal organizations with the academic success

of those who do not belong to fraternal organizations. The reason for the comparison centers on the recent concern that fraternity members, on the average, perform at a lower academic level than the nonfraternal students. Cumulative grade–point average is used to measure academic success. Random samples of size 40 are taken from each population. The data is given in the following table.

Sample	Number	Mean	Standard Deviation
Fraternity members	40	2.03	0.69
Nonmembers	40	2.21	0.59

Complete a hypothesis test using $\alpha = 0.05$. Assume grade–point averages for both groups are approximately normally distributed.

Calculator Solution:

(S1) Both populations are assumed to be approximately normally distributed. The samples are independent and no assumptions are made about the population variances. The test statistic is t^* .

(S2) The four parts of the hypothesis test are:

$$H_0: \mu_1 - \mu_2 = 0 (\geq)$$

$$H_a: \mu_1 - \mu_2 \leq 0$$

$$\text{TS: } t^* = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$\text{CR: } t^* \leq -t(\text{df}, \alpha/2) = -t(76, 0.025) \approx -1.99$$

(S3) Select **[STAT]**; TESTS; 2-SampTTest. Highlight **Stats** and enter the summary statistics for each group. Select the alternative hypothesis and highlight **No** for pooled variance (no assumptions about the population variances). Highlight **Calculate** and press **[ENTER]** to conduct the hypothesis test (Figure 10.10).

(S4) The hypothesis test results are displayed on the Home Screen (Figure 10.11).

(S5) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $t^* = -1.2645$, does not lie in the critical region (equivalently, $P = .1049 > .05$). There is no evidence to suggest fraternity members perform at a lower academic level than nonmembers.

Note: The **Draw** results are shown in Figure 10.12.

◇ □

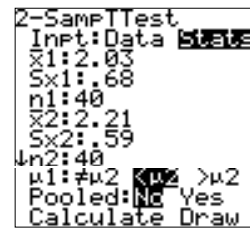


Figure 10.10:
2-SampTTest input screen.

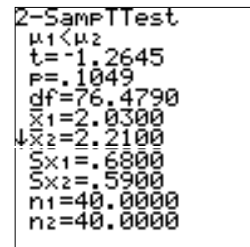


Figure 10.11:
Hypothesis test results.

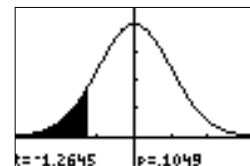


Figure 10.12:
Hypothesis test Draw results.

Note:

- (N1) Suppose both populations are assumed to be normally distributed, the samples are independent, and the population variances are assumed *equal*. A $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ has endpoints

$$(\bar{x}_1 - \bar{x}_2) \pm t(n_1 + n_2 - 2, \alpha/2) \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where s_p is the *pooled* estimate of the common standard deviation,

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

The calculator function **2-SampTInt** (in the **STAT**; TESTS menu) may be used to construct this interval estimate. Highlight **Yes** for pooled variance.

The hypothesis test procedure:

$$H_0: \mu_1 - \mu_2 = \Delta_0$$

$$H_a: \mu_1 - \mu_2 > \Delta_0, \quad \mu_1 - \mu_2 < \Delta_0, \quad \mu_1 - \mu_2 \neq \Delta_0$$

$$\text{TS: } t^* = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{CR: } t^* \geq t(n_1 + n_2 - 2, \alpha), \quad t^* \leq -t(n_1 + n_2 - 2, \alpha), \\ |t^*| \geq t(n_1 + n_2 - 2, \alpha/2)$$

The calculator function **2-SampTTest** may be used to conduct this hypothesis test. Highlight **Yes** for pooled variance.

- (N2) Suppose

- (A1) Both populations are assumed normally distributed, the samples are independent, and both population variances are known, or

- (A2) Both samples are large, the samples are independent, and both population variances are known.

A $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ has endpoints

$$(\bar{x}_1 - \bar{x}_2) \pm z(\alpha/2) \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

The calculator function **2-SampZInt** (in the **STAT**; TESTS menu) may be used to construct this interval. This function accepts data stored in lists or summary statistics.

The hypothesis test procedure:

$$H_0: \mu_1 - \mu_2 = \Delta_0$$

$$H_a: \mu_1 - \mu_2 > \Delta_0, \quad \mu_1 - \mu_2 < \Delta_0, \quad \mu_1 - \mu_2 \neq \Delta_0$$

$$\text{TS: } z^* = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

The calculator function `2-SampZTest` (in the `STAT`; TESTS menu) may be used to conduct this hypothesis test. This function accepts data stored in lists or summary statistics. \diamond

The TI-83 Plus also has a built-in function for constructing a confidence interval for the difference between two proportions and a hypothesis test function for comparing two proportions. The following additional notation will be used to describe and illustrate these procedures.

(N1) p_1, p_2 : population proportions for population 1 and 2, respectively.

(N2) X_1, X_2 : number of successes in sample 1 and 2, respectively.

(N3) p'_1, p'_2 : sample proportions for sample 1 and 2, respectively.

(N4) $q'_1 = 1 - p'_1, \quad q'_2 = 1 - p'_2$.

Suppose random samples are selected independently from two populations and that each sample is large enough so that the distribution for the difference between the two sample proportions is approximately normal. A $100(1 - \alpha)\%$ confidence interval for $p_1 - p_2$ has endpoints

$$(p'_1 - p'_2) \pm z(\alpha/2) \sqrt{\frac{p'_1 q'_1}{n_1} + \frac{p'_2 q'_2}{n_2}}.$$

The TI-83 Plus function `2-PropZInt` may be used to construct this interval estimate.

The hypothesis test procedure:

$$H_0: p_1 - p_2 = 0$$

$$H_a: p_1 - p_2 > 0, \quad p_1 - p_2 < 0, \quad p_1 - p_2 \neq 0$$

$$\text{TS: } z^* = \frac{p'_1 - p'_2}{\sqrt{(p'_p)(q'_p) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

where p'_p is an estimate of the *common* population proportion:

$$p'_p = \frac{X_1 + X_2}{n_1 + n_2}, \quad q'_p = 1 - p'_p.$$

The calculator function `2-PropZTest` may be used to conduct this hypothesis test.

Note: If the hypothesized difference between population proportions is nonzero, $H_0 : p_1 - p_2 = \Delta_0 \neq 0$, the test statistic becomes

$$\text{TS: } z^* = \frac{(p'_1 - p'_2) - \Delta_0}{\sqrt{\frac{p'_1 q'_1}{n_1} + \frac{p'_2 q'_2}{n_2}}}.$$

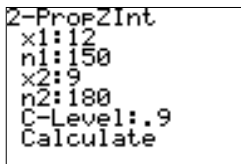
There is no built-in calculator function to conduct this hypothesis test. \diamond

Example: The proportions of defective parts produced by two machines were compared, and the following data was collected.

Machine	Sample Size	Number Defective
1	150	12
2	180	9

Find a 90% confidence interval for the difference in population proportions of defective parts, $p_1 - p_2$.

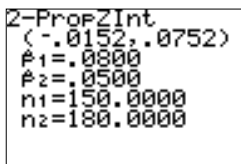
Calculator Solution:



```

2-PropZInt
x1:12
n1:150
x2:9
n2:180
C-Level:.9
Calculate
  
```

Figure 10.13:
2-PropZInt input
screen.



```

2-PropZInt
(-.0152,.0752)
p1=.0800
p2=.0500
n1=150.0000
n2=180.0000
  
```

Figure 10.14:
Confidence interval for
 $p_1 - p_2$.

(S1) The samples are assumed independent and each sample size is large enough so that the distribution for the difference between the two sample proportions is approximately normal. The confidence interval is based on the standard normal distribution.

(S2) Select `[STAT]`; `TESTS`; `2-PropZInt`. Enter the number of successes and sample size for each group, and the level of confidence. Highlight `Calculate` and press `[ENTER]` (Figure 10.13).

(S3) The confidence interval and summary statistics are displayed on the Home Screen (Figure 10.14).

Note: The calculator uses \hat{p}_1 and \hat{p}_2 to denote the sample proportions. \diamond

(S4) $(-.0152, .0752)$ is a 90% confidence interval for the difference in population proportions of defective parts. \square

Example: A recent study reported that smoking boosts death risk for diabetics. The death risk is increased more for women than men. Suppose

as a follow-up study the smoking rates for male and female diabetics were investigated. The following data was obtained.

Gender	n	Number Who Smoke
Male	500	215
Female	500	170

Test the hypothesis that the smoking rate (proportion of smokers) is higher for males than for females. Use $\alpha = 0.05$.

Calculator Solution:

- (S1) The samples are assumed independent and each sample size is large enough so that the distribution for the difference between the two sample proportions is approximately normal. The population proportions are assumed equal. The test statistic is z^* .
- (S2) The four parts of the hypothesis test are:

$$H_0: p_1 - p_2 = 0 (\leq)$$

$$H_a: p_1 - p_2 > 0$$

$$\text{TS: } z^* = \frac{p'_1 - p'_2}{\sqrt{(p'_p)(q'_p) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\text{CR: } z^* \geq z(\alpha) = z(0.05) = 1.6449$$

- (S3) Select **[STAT]**; **TESTS**; **2-PropZTest**. Enter the number of successes and sample size for each group. Highlight the upper-tailed alternative hypothesis and **Calculate** the results (Figure 10.15).
- (S4) The hypothesis test results are displayed on the Home Screen (Figure 10.16).
- (S5) Decision: reject H_0 .

Conclusion: The value of the test statistic, $z^* = 2.9245$, lies in the critical region (equivalently, $P = .0017 < .05$). There is evidence to suggest the proportion of smokers is higher for males than for females.

Note:

- (N1) The **Draw** results are shown in Figure 10.17.
- (N2) On the **Calculate** results screen, \hat{p} is the pooled estimate of the common population proportion. $\diamond \square$

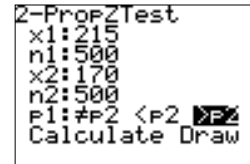


Figure 10.15:
2-PropZTest input screen.

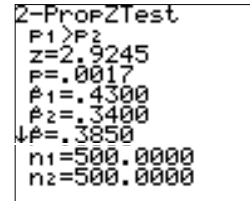


Figure 10.16:
Hypothesis test results.

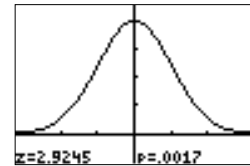


Figure 10.17:
Hypothesis test Draw results.

When comparing two populations, it is reasonable to consider the two most fundamental distribution characteristics: center and variability. If both populations are normal and samples are selected independently, a

hypothesis test uses the ratio of sample variances and is based on an F distribution. The built-in calculator function **2-SampFTest** may be used to compare two population variances (or standard deviations).

The hypothesis test procedure for comparing population variances:

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 > \sigma_2^2, \quad \sigma_1^2 < \sigma_2^2, \quad \sigma_1^2 \neq \sigma_2^2$$

$$\text{TS: } F^* = S_1^2/S_2^2$$

$$\text{CR: } F^* \geq F(n_1 - 1, n_2 - 1, \alpha), \quad F^* \leq F(n_1 - 1, n_2 - 1, 1 - \alpha),$$

$$F^* \leq F(n_1 - 1, n_2 - 1, 1 - \alpha/2) \quad \text{or} \quad F^* \geq F(n_1 - 1, n_2 - 1, \alpha/2)$$

where $F(\nu_1, \nu_2, \alpha)$ denotes a critical value form an F distribution with ν_1 and ν_2 degrees of freedom.

Example: A recent study was conducted to compare the variability in male and female systolic blood pressures. Random samples of 16 men and 13 women were obtained and the systolic blood pressures are given in the table below.

	Systolic Blood Pressure							
Men	120	140	118	112	145	114	130	114
	124	125	130	100	120	108	112	122
Women	122	102	118	126	108	130	104	116
	102	122	120	118	130			

Is there any evidence to suggest the variance in male systolic blood pressure is greater than the variance in female systolic blood pressure? Complete the hypothesis test using $\alpha = .01$ and assume both populations are normally distributed.

Calculator Solution:

(S1) Both populations are assumed normal and the samples were selected independently. The test statistic is F^* .

(S2) The four parts of the hypothesis test are:

$$H_0: \sigma_1^2 = \sigma_2^2 (\leq)$$

$$H_a: \sigma_1^2 > \sigma_2^2$$

$$\text{TS: } F^* = S_1^2/S_2^2$$

$$\text{CR: } F^* \geq F(n_1 - 1, n_2 - 1, \alpha) = F(15, 12, .01) = 4.01$$

(S3) Enter the systolic blood pressure for men in the list **SBPM** and the systolic blood pressure for women in the list **SBPW**.

- (S4) In the **STAT**; TESTS menu, select 2-SampFTest. Highlight Data, enter the list names for each data set, and set the frequency for each data set to 1. Select the one-sided alternative hypothesis and Calculate the results (Figure 10.18).

```

2-SampFTest
Inpt:DATA Stats
List1:SBPM
List2:SBPW
Frc1:1
Frc2:1
σ1:≠σ2 <σ2 >σ2
Calculate Draw

```

Figure 10.18:
2-SampFTest input screen.

- (S5) The hypothesis test results are displayed on the Home Screen (Figure 10.19).

```

2-SampFTest
σ1>σ2
F=1.4847
P=.2570
Sx1=11.5636
Sx2=9.4900
x̄1=120.8750
x̄2=115.6667
n1=16.0000
n2=12.0000

```

Figure 10.19:
Hypothesis test results.

- (S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $F^* = 1.4847$, does not lie in the critical region (equivalently, $P = .2570 > .01$). There is no evidence to suggest the variance in systolic blood pressure is higher for men than women.

Note: The Draw results are shown in Figure 10.20. $\diamond \square$

Note:

- (N1) In the **DISTR**; DISTR menu the function **Fpdf** returns a value of the probability density function for an F distribution with specified (ν_1, ν_2) degrees of freedom; calculator function with arguments: **Fpdf**(x, ν_1, ν_2). Figure 10.20 shows the graph of the probability density function for an F random variable with 8 and 10 degrees of freedom (WINDOW settings: $[0, 5] \times [0, .8]$).

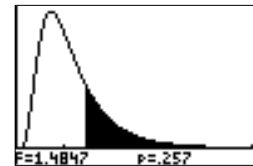


Figure 10.20:
Hypothesis test Draw results.

- (N2) The function **Fcdf** computes the probability of selecting a value between a lower (a) and upper (b) bound for an F distribution with specified (ν_1, ν_2) degrees of freedom; function with arguments: **Fcdf**(a, b, ν_1, ν_2). Figure 10.22 shows the probability of selecting a value between 1 and 2.5 from an F distribution with 4 and 15 degrees of freedom is .3513.

```

Fcdf(1,2.5,4,15)
.3513

```

Figure 10.22: Use of **Fcdf** to compute probability.

- (N3) In the **DISTR**; DRAW menu the function **ShadeF** draws the probability density function for an F distribution with specified (ν_1, ν_2) degrees of freedom, shades the area between a lower (a) and upper (b) bound, and computes the shaded area; function with arguments: **ShadeF**(a, b, ν_1, ν_2). Figure 10.23 illustrates the use of **ShadeF** to visualize the probability found in note (N2). \diamond

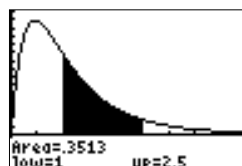


Figure 10.23: Use of **ShadeF** to illustrate (and compute) probability.

Notes:

Chapter 11

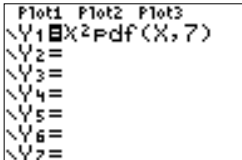
Applications of Chi-Square

The chi-square distribution is used to test hypotheses concerning enumerated data. This distribution is nonnegative in value, not symmetrical, and skewed to the right. A chi-square distribution is completely determined by the number of degrees of freedom.

Recall there are three built-in functions that involve the chi-square probability function and cumulative probability function (introduced in Chapter 9).

- (R1) $\chi^2\text{pdf}(x, \nu)$ (in the **DISTR**; DISTR menu): returns a value of the probability function for a chi-square distribution with ν degrees of freedom. Figure 11.1 shows a use of this function in the Y= editor. Figure 11.2 shows the graph of the probability function for a chi-square random variable with 7 degrees of freedom (WINDOW settings: $[0, 25] \times [0, .13]$).
- (R2) $\chi^2\text{cdf}(a, b, \nu)$ (in the **DISTR**; DISTR menu): returns the probability of selecting a value between a and b for a chi-square distribution with ν degrees of freedom. Figure 11.3 shows the probability of selecting a value between 3 and 7 from a chi-square distribution with 12 degrees of freedom is .1379.
- (R3) $\text{Shade}\chi^2(a, b, \nu)$ (in the **DISTR**; DRAW menu): draws the probability density function for a chi-square distribution with ν degrees of freedom, shades and computes the area between a and b . Figure 11.4 illustrates the use of $\text{Shade}\chi^2$ (on the Home Screen) to visualize the probability found in note (R2) and Figure 11.5 shows the resulting graph (WINDOW settings: $[0, 35] \times [-.0225, .09]$).

Note! As the number of degrees of freedom increases, the chi-square



```
Plot1 Plot2 Plot3
Y1=X^2pdf(X,7)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

Figure 11.1: Y= setup to draw a chi-square probability function.

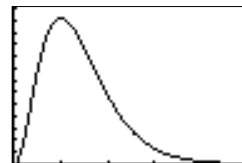
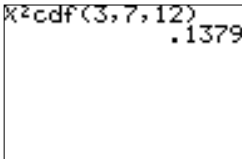


Figure 11.2: Chi-square probability function.



```
X^2cdf(3,7,12)
.1379
```

Figure 11.3: Use of $\chi^2\text{cdf}$ to compute probability.

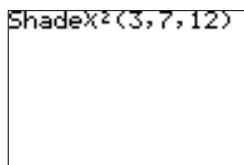


Figure 11.4: $\text{Shade}\chi^2$ use on the Home Screen.

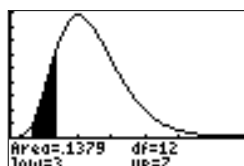


Figure 11.5: Resulting probability function, shaded area, and computed probability .

distribution approaches a symmetric (normal) distribution. Figure 11.5 suggests this and further graphs, with larger number of degrees of freedom, will (graphically) support this approximation result. \diamond

A **multinomial experiment** has the following characteristics:

- (C1) It consists of n identical independent trials.
- (C2) The outcome of each trial falls into exactly one of k possible cells.
- (C3) There is a probability associated with each cell, p_i , and these probabilities remain constant during the experiment.
- (C4) The experiment will result in a set of observed frequencies, O_1, O_2, \dots, O_k , where each O_i is the number of times a trial outcome falls into cell i . $O_1 + O_2 + \dots + O_k = n$.

The testing procedure for multinomial experiments, or **goodness-of-fit test**, is designed to test a hypothesis about the cell probabilities. The test statistic compares the observed frequencies with the expected frequencies and has a chi-square distribution.

Goodness-of-fit hypothesis test:

H_0 : $p_1 = p_2 = \dots = p_k$
 (The proportion of observations in each cell is independent of cell; all proportions are equal.)

H_a : $p_i \neq p_j$ for some $i \neq j$
 (The number of observations in each cell depends on the cell; not all proportions are equal.)

$$\text{TS: } \chi^{2*} = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$$

$$\text{CR: } \chi^{2*} \geq \chi^2(k - 1, \alpha)$$

The expected number of observations in cell i is $E_i = n \cdot p_i$. The calculator program **GOODNESS** may be used to conduct this hypothesis test. There is also an option in this program for *unequal* expected frequencies.

Example: College students have regularly insisted on freedom of choice when registering for courses. During a recent semester there were seven sections of an introductory statistics class. The classes were scheduled to meet at various times with a variety of instructors. The table below shows the number of students who selected each of the seven sections.

Section	1	2	3	4	5	6	7
Number of Students	18	12	25	23	8	19	14

Is there any evidence to indicate students had a preference for certain

sections, or does the data suggest each section was equally likely to be selected? Use $\alpha = 0.05$.

Calculator Solution:

(S1) There are 7 cells, or class sections. We are interested in knowing if students had a preference for one section over the others or are all sections preferred equally. A goodness-of-fit test with equal expected frequencies is appropriate.

(S2) The four parts of the hypothesis test are:

H_0 : $p_1 = p_2 = \cdots = p_7 (= 1/7)$
There was no preference shown (equally distributed).

H_a : $p_i \neq p_j$ for some $i \neq j$
There was a preference shown (not equally distributed).

$$\text{TS: } \chi^{2*} = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$$

$$\text{CR: } \chi^{2*} \geq \chi^2(k - 1, \alpha) = \chi^2(6, 0.05) = 12.6$$

(S3) Enter the observed frequencies in the list **CLASS**. Execute the program **GOODNESS**.

(S4) At the first program menu, select equal expected frequencies (Figure 11.6). Enter the list containing the observed frequencies (Figure 11.7).

(S5) The hypothesis test results are displayed on the Home Screen (Figure 11.8).

(S6) Decision: reject H_0 .

Conclusion: The value of the test statistic, $\chi^{2*} = 12.9412$, lies in the critical region (equivalently, $P = .0440 < .05$). There is evidence to suggest students had a preference for one class over the others. \square

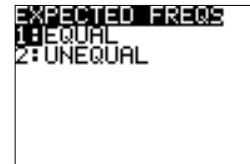


Figure 11.6: Goodness-of-fit test: equal expected frequencies.

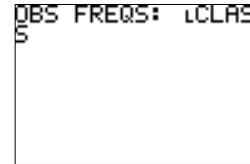


Figure 11.7: Observed frequencies list.

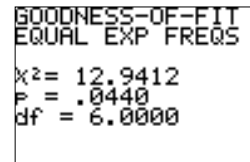


Figure 11.8: Hypothesis test results.

If the expected frequencies are not all equal, the hypothesis test is still based on a chi-square distribution. If the expected frequencies are not known, they are computed from the known expected proportions. The test statistic is the same.

Example: A recent article about the vacation home market reported the percentage of people who preferred each unit size (percent preferring). In a follow-up survey, 300 individuals were asked their vacation home unit size preference (number preferring). The article and survey results are given in the table below. Does the distribution of preferences in the follow-up study differ from the distribution reported in the article? Use $\alpha = 0.01$.

STUDY	UNIT	-----	1
75.0000	.1820		
60.0000	.1820		
105.00	.4040		
45.0000	.1820		
15.0000	.0500		
-----	-----		
STUDY(1)=75			

Figure 11.9: Observed frequencies and expected proportions.

EXPECTED FREQS
1: EQUAL
2: UNEQUAL

Figure 11.10: Unequal expected frequencies option.

KNOWN EXPECTED
1: FREQUENCIES
2: PROPORTIONS

Figure 11.11: Known expected proportions option.

OBS FREQS: LSTUD
EXP PROPS: LUNIT

Figure 11.12: List entries.

GOODNESS-OF-FIT
UNEQUAL EXP FREQ
$\chi^2 = 12.0093$
$P = .0173$
$df = 4.0000$

Figure 11.13: Hypothesis test results.

Unit Size	Percent Preferring	Number Preferring
Studio/efficiency	18.2	75
1 bedroom	18.2	60
2 bedrooms	40.4	105
3 bedrooms	18.2	45
Over 3 bedrooms	5.0	15

Calculator Solution:

(S1) We are interested in knowing if there is a difference in the proportions reported in the article and the proportions in the follow-up study. The expected frequencies (proportions) are unequal. A goodness-of-fit test is appropriate.

(S2) The four parts of the hypothesis test are:

H_0 : The proportion of people in the follow-up study preferring each unit size are as given in the article.

H_a : The proportions are not as given in the article.

$$\text{TS: } \chi^{2*} = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$$

$$\text{CR: } \chi^{2*} \geq \chi^2(k - 1, \alpha) = \chi^2(4, 0.01) = 13.3$$

(S3) Enter the observed frequencies in the list STUDY and the expected proportions in the list UNIT (Figure 11.9).

Note: The program GOODNESS uses the lists EXP, OBS, and PROPS. \diamond

(S4) Execute the program GOODNESS. Select unequal frequencies (Figure 11.10) and known expected proportions (Figure 11.11).

(S5) Enter the observed frequency list and the expected proportion list (Figure 11.12). The hypothesis test results are displayed on the Home Screen (Figure 11.13).

(S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $\chi^{2*} = 12.0093$, does not lie in the critical region (equivalently, $P = .0173 > .01$). At the $\alpha = 0.01$ level, there is not enough evidence to suggest the proportions in the follow-up study differ from those given in the article. \square

A **contingency table** is an arrangement of data into a two-way classification. Each observed item is simultaneously classified by two variables or traits. A hypothesis test based on the chi-square distribution is used to determine whether the two variables are independent or dependent.

Suppose a contingency table contains r rows and c columns. The expected frequency at the intersection of the i th row and the j th column is given by

$$E_{i,j} = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}} = \frac{R_i \times C_j}{n}.$$

Hypothesis test for independence of two variables
(contingency table analysis):

H_0 : The two variables are independent.

H_a : The two variables are dependent.

$$\text{TS: } \chi^{2*} = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$$

$$\text{CR: } \chi^{2*} \geq \chi^2((r - 1)(c - 1), \alpha)$$

The built-in calculator function χ^2 -Test (in the **STAT**; TESTS menu) may be used to test the null hypothesis of independence of two variables. The observed frequencies must be stored in a TI-83 Plus matrix. The calculator function computes the expected frequencies and stores them in another user-specified matrix.

Example: The manager of an assembly process would like to determine whether the number of defective articles manufactured depends on the day of the week the articles are produced. The following information was collected.

Day of Week	Nondefective	Defective
Monday	185	15
Tuesday	190	7
Wednesday	195	12
Thursday	187	10
Friday	175	18

Is there any evidence to suggest the number of defective articles is dependent on the day of the week? Use $\alpha = 0.05$.

Calculator Solution:

(S1) We would like to know if the two variables, number of defectives and day of the week, are related. A chi-square test based on the contingency table is appropriate in order to test for dependence.

(S2) The four parts of the hypothesis test are:

H_0 : Number of defectives and day of the week are independent.

H_a : Number of defectives and day of the week are not independent.

MATRIX[A] 5 × 2	
185	15
190	7
195	12
187	10
175	18

Figure 11.14:
Contingency table in
[A].

χ ² -Test
Observed: [A]
Expected: [B]
Calculate Draw

Figure 11.15: χ^2 -Test
input screen.

χ ² -Test
χ ² =6.6437
P=.1560
df=4.0000

Figure 11.16:
Hypothesis test results.

MATRIX[B] 5 × 2	
187.5	12.5
184.7	12.3
184.1	12.9
184.7	12.3
181.0	12.0

Figure 11.17: Expected
frequencies.

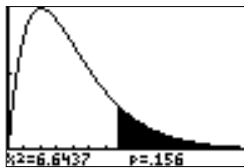


Figure 11.18:
Hypothesis test Draw
results.

$$\text{TS: } \chi^{2*} = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$$

$$\text{CR: } \chi^{2*} \geq \chi^2((r - 1)(c - 1), \alpha) = \chi^2(4, 0.05) = 9.49$$

- (S3) Enter the contingency table in the matrix [A] (Figure 11.14).
- (S4) Select **[STAT]**; TESTS; χ^2 -Test. Enter the matrix containing the contingency table (**Observed**) and specify a matrix for the expected frequencies (Figure 11.15). **Calculate** the results.
- (S5) The hypothesis test results are displayed on the Home Screen (Figure 11.16) and the expected frequencies are stored in the specified matrix (Figure 11.17).
- (S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $\chi^{2*} = 6.6437$, does not lie in the critical region (equivalently, $P = .1560 > .05$). There is no evidence to suggest number of defectives and day of the week are dependent.

Note: The **Draw** results are shown in Figure 11.18. $\diamond \square$

Note:

- (N1) A **test of homogeneity** is another type of contingency table problem. It is used when one of the two variables is controlled by the experimenter so that the row (or column) totals are predetermined. The test statistic is the same and the calculator function χ^2 -Test may be used to conduct this test.
- (N2) The expected frequency in each cell should be at least 5. If several cells have small expected frequencies, then cells might be combined; collapse two or more rows/columns into one.
- (N3) No cell should have expected frequency less than 1. Another solution to small expected frequencies is to increase the sample size. \diamond

Chapter 12

Analysis of Variance

Many experiments are conducted to determine the effect of different levels of a test factor on a response variable. **Analysis-of-variance** (ANOVA) is a statistical technique used to test equality of population means. In order to compare the means of the test factor, a measure of the variation between the levels, MS(factor), is compared to a measure of the variation within the levels, MS(error). If the quantity MS(factor) is significantly larger than MS(error), then there is evidence to suggest the means for each of the factor levels are not all the same.

The TI-83 Plus built-in function **ANOVA** may be used to compare the means in up to 20 populations, or factor levels. The assumptions for ANOVA are:

- (A1) Each population is normally distributed.
- (A2) Each population has the same variance, σ^2 .
- (A3) The samples are selected independently from each population.

The hypothesis test for a one-way ANOVA is based on an F distribution that compares the variation *between* the samples to the variance *within* the samples. The standard ANOVA table is given below.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Test Statistic
Factor	SS(factor)	$c - 1$	MS(factor)	$F^* = \frac{\text{MS}(\text{factor})}{\text{MS}(\text{error})}$
Error	SS(error)	$n - c$	MS(error)	
Total	SS(total)	$n - 1$		

Notation:

SS(factor) = Sum of squares due to factors (*between* sample variation).

SS(error) = Sum of squares due to error (*within* sample variation).

SS(total) = Sum of squares due to total (*total* variation).

MS(factor) = Mean square due to factors.

$$= \text{SS}(\text{factor}) / (c - 1)$$

MS(error) = Mean square due to error.

$$= \text{SS}(\text{error}) / (n - c)$$

c = Number of factors, or populations, being studied.

n = Total number of observations.

ANOVA hypothesis test:

H_0 : The c factor (population) means are equal.

$$(\mu_1 = \mu_2 = \cdots = \mu_c)$$

H_a : At least two factor (population) means are unequal.

$$(\mu_i \neq \mu_j \text{ for some } i \neq j)$$

TS: $F^* = \text{MS}(\text{factor}) / \text{MS}(\text{error})$

CR: $F^* \geq F(c - 1, n - c, \alpha)$

Note:

(N1) If the null hypothesis is true (all means equal), then the test statistic, F^* , has an F distribution with $c - 1$ and $n - c$ degrees of freedom.

(N2) The mean square due to error, MS(error), is an unbiased estimate of the common variance σ^2 .

(N3) Sample sizes do not have to be equal. However, if all groups have the same number of observations, the probability of a type II error is minimized. \diamond

Example: A recent study compared the number of calories in certain kinds of diet snack foods. The researcher randomly selected 50 snacks and divided them into four categories: crunchy choices, salty sensations, creamy concoctions, and sinless sweets. The calorie content for each snack was measured and the results are given in the table below. Is there any evidence to suggest a difference in the mean calorie content for the four different snack food categories? Use $\alpha = .01$.

Calculator Solution:

(S1) There are four factors (populations) of interest (calorie content of the four snack foods), all assumed to be normally distributed with equal variances, and the samples were selected independently. A

one-way analysis-of-variance is appropriate.

Crunchy Choices	Snack Food Category		
	Salty Sensations	Creamy Concoctions	Sinless Sweets
89	100	99	90
99	32	97	99
91	60	65	94
76	65	79	91
90	100	99	100
90	70	90	54
97	5	90	87
52	14	100	88
93	52	50	96
80	59	94	50
82		100	70
90		70	90
			75
			100
			70
			88

(S2) The four parts of the hypothesis test are:

H_0 : The four snack food categories have equal mean calorie content.

$$\mu_1 = \mu_2 = \mu_3 = \mu_4$$

H_a : At least two of the categories have different mean calorie content. $\mu_i \neq \mu_j$ for some $\mu_i \neq \mu_j$

TS: $F^* = MS(\text{factor})/MS(\text{error})$

CR: $F^* \geq F(c - 1, n - c, .01) = F(3, 46, .01) = 4.24$

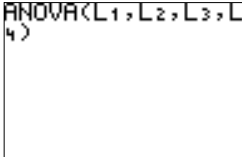
(S3) Enter the observations for each category in the calculator lists L1 - L4. (Named lists may also be used.)

(S4) From the Home Screen, select ANOVA (in the **STAT**; TESTS menu). Enter the arguments, the lists containing the observations, for this function separated by commas (Figure 12.1). Press **ENTER**.

(S5) The hypothesis test results are displayed on the Home Screen (Figure 12.2).

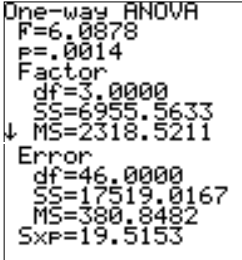
(S6) Decision: reject H_0 .

Conclusion: The value of the test statistic, $F^* = 6.0878$, lies in the critical region (equivalently, $P = .0014 < .01$). There is evidence to suggest at least two of the mean calorie contents are unequal.



```
ANOVA(L1,L2,L3,L4)
```

Figure 12.1: Calculator ANOVA function.



```
One-way ANOVA
F=6.0878
P=.0014
Factor
df=3.0000
SS=6955.5633
MS=2318.5211
Error
df=46.0000
SS=17519.0167
MS=380.8482
SxP=19.5153
```

Figure 12.2: ANOVA results.

Note:

- (N1) The ANOVA table (given below) may be constructed from the calculator output shown in Figure 12.2.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Test Statistic
Factor	6955.56	3	2318.52	6.09
Error	17519.02	46	380.85	
Total	24474.58	49		

The value of S_{xp} is the pooled estimate of the common standard deviation.

- (N2) The conclusion in this example is that at least two of the mean calorie contents are unequal. The next natural question is which pairs of means are unequal. Other statistical techniques, for example, Tukey's procedure or Duncan's multiple-range test, may be used to isolate pairwise differences.
- (N3) Other statistical procedures, for example, Bartlett's test, may be used to check the assumption of equal variances. $\diamond \square$

Chapter 13

Linear Correlation and Regression Analysis

Recall:

- (R1) If bivariate data consists of two quantitative variables, the values are often expressed mathematically as ordered pairs (x, y) . The first value, x , is the input, or independent, variable, and the second value, y , is the output, or dependent, variable. This bivariate data may be presented graphically on a scatter diagram: a plot of all ordered pairs of bivariate data on a coordinate axis system. The input variable x is plotted on the horizontal axis, and the output variable y is plotted on the vertical axis. A scatter diagram, or scatter plot, is a TI-83 Plus built-in statistical plot.
- (R2) Suppose $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ represent n pairs of observations on two variables. The coefficient of linear correlation, or Pearson's product moment correlation coefficient, r , is a numerical measure of the strength of the linear relationship between the two variables. The formula for r is

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{(n - 1)s_x s_y} = \frac{SS(xy)}{\sqrt{SS(x)}\sqrt{SS(y)}}.$$

Chapter 3 contains examples that illustrates these two concepts.

Another measure of linear dependency is the covariance. The **covariance** of x and y is defined as the sum of the products of the distances of all values of x and y from the centroid divided by $n - 1$:

$$\text{covar}(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n - 1}.$$

Note:

- (N1) The covariance is positive if the scatter diagram is dominated by points to the upper right and to the lower left of the centroid, (\bar{x}, \bar{y}) . If the majority of points are in the upper left and lower right sections relative to the centroid, the covariance is negative.
- (N2) The sign of the covariance is always the same as the sign of the slope of the regression line.
- (N3) The covariance does not have a standardized unit of measure. A *standardized* measure of linear dependency is the coefficient of linear correlation (as defined above, and also given by):

$$r = \frac{\text{covar}(x, y)}{s_x \cdot s_y}.$$

Example: Professional golfers are commonly rated on the basis of how much money they earn while playing on the tour. They are also given a world point ranking that considers their performances for the past two years, with an emphasis on the last year. Money winnings in 1998 through August of that year and the world point rankings for 16 players are shown in the table below.

Player	Money Earnings (\$)	Points
David Duval	1,668,678	9.42
Mark O'Meara	1,523,295	9.51
Fred Couples	1,495,698	7.50
Tiger Woods	1,388,542	11.82
Jim Furyk	1,292,346	6.59
Justin Leonard	1,253,129	7.14
Lee Janzen	1,052,622	5.45
Mark Calcavecchia	910,254	5.51
Scott Hoch	878,623	5.76
Phil Mickelson	873,477	7.61
Jesper Parnevik	857,956	5.73
Tom Watson	822,385	5.47
Davis Love III	815,766	10.59
Tom Lehman	812,114	5.91
Vijay Singh	745,661	6.25
Nick Price	484,737	7.94

EARN	POINT	-----	1
1668678	9.4200		
1523295	9.5100		
1495698	7.5000		
1388542	11.8200		
1292346	6.5900		
1253129	7.1400		
1052622	5.4500		
910254	5.5100		
878623	5.7600		
873477	7.6100		
857956	5.7300		
822385	5.4700		
815766	10.5900		
812114	5.9100		
745661	6.2500		
484737	7.9400		
EARN(15)=745661			

Figure 13.1: Money earnings and points data.

Construct a scatter diagram for this data. Calculate the covariance and the linear correlation coefficient between money earnings and the world point rankings. What conclusions might you draw from these results?

Calculator Solution:

- (S1) Enter the money earnings data in the list **EARN** and the world point rankings in the list **POINT** (Figure 13.1).

- (S2) Set up Plot1 (STATPLOT; STATPLOTS; Plot1). Select On, Type scatter plot (the first plot type), set Xlist to EARN, Ylist to POINT, and select the desired Mark (Figure 13.2).
- (S3) Select appropriate WINDOW settings (Figure 13.3) and press GRAPH to display the scatter diagram (Figure 13.4).
- (S4) On the Home Screen, execute the program COVAR. Enter the list containing the money earnings and the list containing the world point rankings (Figure 13.5). The covariance is displayed on the Home Screen (Figure 13.6): $\text{covar}(x, y) = 295797.1412$.
- (S5) On the Home Screen execute DiagnosticOn (from the CATALOG menu). Select LinReg(a+bx) (in the STAT; CALC menu), and enter the lists EARN and POINT (Figure 13.7). The results are displayed on the Home Screen (Figure 13.8): $r = .4405$. The scatter diagram and the coefficient of linear correlation indicate only a weak positive linear relationship between money earnings and world point rankings. \square



Figure 13.2: Plot1 settings.

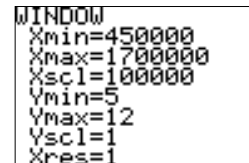


Figure 13.3: WINDOW settings.

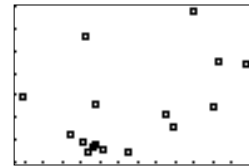


Figure 13.4: Scatter diagram.

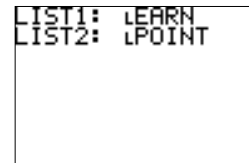


Figure 13.5: COVAR input.

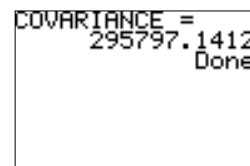


Figure 13.6: Covariance between EARN and POINT.

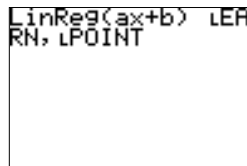
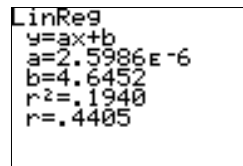
Figure 13.7: Use of LinReg(a+bx) to compute r .

Figure 13.8: LinReg(a+bx) results.

Suppose the set (x, y) of ordered pairs forms a random sample and the y -values at each x have a normal distribution. The sample linear correlation coefficient r is an estimate of the population linear correlation coefficient ρ and simple inferences are based on a t distribution. Critical values for r are presented in Appendix B of the main text.

Hypothesis test about a population correlation coefficient ρ :

$$H_0: \rho = 0$$

$$H_a: \rho > 0, \quad \rho < 0, \quad \rho \neq 0$$

$$\text{TS: } t^* = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}}$$

$$\text{CR: } t^* \geq t(n-2, \alpha), \quad t^* \leq -t(n-2, \alpha), \quad |t^*| \geq t(n-2, \alpha/2)$$

If the observations are from a bivariate normal distribution, a confidence interval for ρ may be constructed and a more general hypothesis test is based on the standard normal distribution. Confidence belts for r are presented in Appendix B of the main text.

Hypothesis test about ρ (bivariate normal distribution assumed):

$$H_0: \rho = \rho_0$$

$$H_a: \rho > \rho_0, \quad \rho < \rho_0, \quad \rho \neq \rho_0$$

$$\text{TS: } z^* = \frac{\sqrt{n-3}}{2} \ln \left[\frac{(1+R)(1-\rho_0)}{(1-R)(1+\rho_0)} \right]$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

The calculator program **CORRTEST** will compute a confidence interval for ρ , conduct a hypothesis test based on a t distribution, or conduct a more general hypothesis test based on a standard normal distribution.

Example: Two measures of a company's performance are its net income (earnings) and the total market value of its stock. Fifteen random companies were selected and their earnings and total market value of stock (in millions of dollars) were recorded. The data is given in the table below.

Earnings	175.2	8.1	71.7	46.5	33.4	74.5	54.9	48.5
Stock	1986	94	645	2360	641	1178	325	895
Earnings	82.5	59.8	15.6	62.4	73.1	85.9	96.4	
Stock	423	1717	165	611	827	947	845	

Construct a scatter diagram, find the linear correlation coefficient, and test for a significant positive correlation between earnings and market value of stock. Use $\alpha = .05$.

Calculator Solution:

- (S1) Enter the earnings data in the list **EARNNS** and the stock data in the list **STOCK**.
- (S2) Choose appropriate **Plot1** and **WINDOW** ($[0, 200] \times [0, 2500]$) settings, and display the scatter diagram (Figure 13.9).
- (S3) Execute **DiagnosticOn** (if not in effect) and execute **LinReg(a+bx)** with the arguments **EARNNS** and **STOCK**. The **LinReg(a+bx)** results are shown in Figure 13.10. The scatter diagram and $r = .4905$ suggest a moderate correlation between earnings and market value of stock.
- (S4) Assume the ordered pairs (earnings, stock) from a random sample and the market value of stock has a normal distribution for each value of earnings. The population parameter of interest is ρ , the linear correlation coefficient between earnings, or net income, and total market value of stock and the hypothesis test is based on a t distribution. Since the hypothesis test is for a significant positive correlation between earnings and market value of stock, the appropriate alternative hypothesis is one-sided upper-tailed.

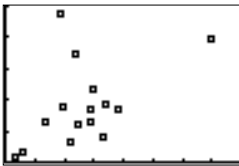


Figure 13.9: Scatter diagram.

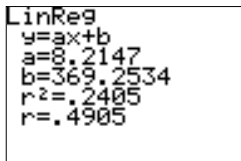


Figure 13.10: Sample correlation coefficient r .

(S5) The four parts of the hypothesis test are:

$$H_0: \rho = 0$$

$$H_a: \rho > 0$$

$$\text{TS: } t^* = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}}$$

$$\text{CR: } t^* \geq t(n-2, \alpha) = t(15, .05) = 1.77$$

(S6) Execute the program CORRTEST. Select DATA at the first program prompt (Figure 13.11).

(S7) Enter the names of the lists containing the data (Figure 13.12). Select the appropriate hypothesis test procedure (Figure 13.13) and alternative hypothesis (Figure 13.14).

(S8) The hypothesis test results are displayed on the Home Screen (Figure 13.15).

(S9) Decision: reject H_0 .

Conclusion: The value of the test statistic, $t^* = 2.0292$, lies in the critical region (equivalently, $P = .0317 < .05$). There is evidence to suggest earnings and stock are positively correlated. \square

Recall from Chapter 3: **Regression analysis** is used to find the equation of the line that best describes the relationship between two variables. The equation for the *best* line, is called the **regression equation** and it may be used to predict a value of the dependent variable based on a selected value of the independent variable.

The linear model used to explain the behavior of linear bivariate data in the population is $Y = \beta_0 + \beta_1 x + \epsilon$. This equation represents the linear relationship between the two variables in a population. β_0 is the y -intercept, β_1 is the slope, and ϵ is the random experimental error.

The regression line computed from the sample data is $\hat{y} = b_0 + b_1 x$: b_0 is an estimate of β_0 and b_1 is an estimate of β_1 . The experimental error is approximated by $e = y - \hat{y}$, the difference between the observed value of y and the predicted value of y , \hat{y} , at a given value of x .

The calculator may be used to employ the **method of least squares** in order to find the *best-fitting* line. The calculation of the coefficients of the regression line and a hypothesis test for a significant regression are built-in calculator functions. Using optional function arguments and plotting features it is possible to view the graph of the regression line and the scatter diagram on the same set of axes, and to obtain predicted values.

```

INPUT:
1:DATA
2:STATS
  
```

Figure 13.11: Program input is data in lists.

```

XLIST: LEARNS
YLIST: LSTOCK
  
```

Figure 13.12: Lists containing the data.

```

PROCEDURE:
1:CONF INTERVAL
2:1 T TEST
3:2 T TEST
  
```

Figure 13.13: Hypothesis test procedure.

```

ALT HYP:
1:RHO > 0
2:RHO < 0
3:RHO ≠ 0
  
```

Figure 13.14: Alternative hypothesis.

```

H0: RHO = 0
HA: RHO > 0
TS = 2.0292
P = .0317
r = .4905
  
```

Figure 13.15: Hypothesis test results.

A hypothesis test for a significant regression is equivalent to testing for a nonzero slope parameter. An estimate of variance of the experimental error, s_e^2 , is required and the test is based on a t distribution. Assume the ordered pairs (x, y) form a random sample and the y -values at each x have a normal distribution.

The hypothesis test procedure for significant regression:

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 > 0, \quad \beta_1 < 0, \quad \beta_1 \neq 0$$

$$\text{TS: } t^* = \frac{b_1}{s_{b_1}} \quad \text{where } s_{b_1}^2 = \frac{s_e^2}{\sum(x - \bar{x})^2}$$

$$\text{CR: } t^* \geq t(n-2, \alpha), \quad t^* \leq -t(n-2, \alpha), \quad |t^*| \geq t(n-2, \alpha/2)$$

A $100(1-\alpha)\%$ confidence interval for the slope parameter β_1 has endpoints

$$b_1 \pm t(n-2, \alpha/2) \cdot s_{b_1}.$$

Example: As part of a study for the graduate school at a large university, a researcher investigated the effect of undergraduate grade point average (GPA) on the graduate record exam (GRE). Fifteen random students were selected and their GPA and GRE scores were recorded. The data is given in the table below.



Figure 13.16:
LinReg(a+bx) setup.

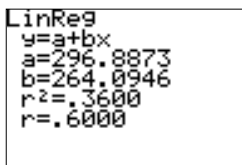


Figure 13.17:
LinReg(a+bx) results.

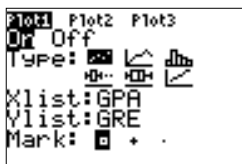


Figure 13.18: Plot1
settings.

GPA (x)	2.30	3.65	3.00	2.75	3.10	2.55	2.50	2.30
GRE (y)	925	1300	1150	1400	900	825	950	1050
GPA (x)	2.90	3.15	3.25	2.00	2.75	2.65	3.13	
GRE (y)	1200	1200	1100	700	850	990	1000	

- (E1) Find the regression coefficients and construct a graph of the regression line and the scatter diagram.
- (E2) Is the regression line significant? (Is there any evidence to suggest $\beta_1 \neq 0$.) Use $\alpha = .05$.
- (E3) If a student has an undergraduate GPA of 3.25, what is the predicted GRE score?

Calculator Solution:

- (S1) Enter the GPA data in the list `GPA` and the GRE data in the list `GRE`.
- (S2) Select the function `LinReg(a+bx)` (in the `STAT`; `CALC` menu) and enter three arguments: independent variable (`GPA`), dependent variable (`GRE`), and a function variable (`Y1`, in the `VARS`; `Y-VARS`; `Function` menu) for storing the regression equation (Figure 13.16). Press `ENTER` to execute this command.

Note:

- (N1) The third argument in $\text{LinReg}(a+bx)$ (calculator function variable) is optional. The regression equation is always stored in the statistics variable RegEQ .
- (N2) If DiagnosticOn is in effect $\text{LinReg}(a+bx)$ will display r and r^2 also. \diamond
- (S3) The slope and the y -intercept for the equation of the line of best fit, r , and r^2 (DiagnosticOn in effect) are displayed on the Home Screen (Figure 13.17). The regression equation is $\hat{y} = 296.8873 + 264.0946x$, $r = .6000$ and $r^2 = .3600$.
- (S4) Select Plot1 settings (Figure 13.18) and appropriate WINDOW settings (Figure 13.19). Press GRAPH to display the scatter diagram and the graph of the equation of the line of best fit (Figure 13.20).

Note: Since the equation of the line of best fit was stored in the function variable $Y1$, the graph of this line will automatically be displayed (along with the scatter diagram). \diamond

- (S5) The four parts of the hypothesis test for a significant regression are:

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 \neq 0$$

$$\text{TS: } t^* = \frac{b_1}{s_{b_1}}$$

$$\text{CR: } |t^*| \geq t(n-2, \alpha/2) = t(13, .025) = 2.160$$

- (S6) Use the calculator built-in function LinRegTTest (in the STAT ; TESTS menu) to test for a significant regression. In the LinRegTTest input screen, enter the independent variable list ($Xlist$:GPA) and the dependent variable list ($Ylist$:GRE), and frequency 1 (since each observation occurs once). The alternative hypothesis is $\beta_1 \neq 0$ or, equivalently, $\rho \neq 0$. The regression equation is always automatically stored in the statistics variable RegEQ and in a function variable if entered. Calculate the results (Figure 13.21).
- (S7) The hypothesis test results are displayed on the Home Screen (Figure 13.22).
- (S8) Decision: reject H_0 .
Conclusion: The value of the test statistic, $t^* = 2.7041$, lies in the critical region (equivalently, $P = .0181 < .05$). There is evidence to suggest a significant regression.
- (S9) The regression equation was stored in the function variable $Y1$ (by $\text{LinReg}(a+bx)$ and written over by LinRegTTest). On the Home Screen, evaluate $Y1$ at 3.25 (Figure 13.23). The regression equation

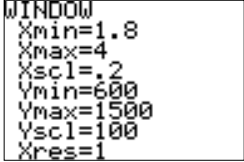


Figure 13.19: WINDOW settings.

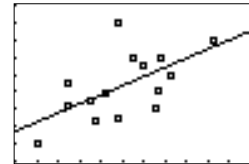


Figure 13.20: Scatter diagram and regression line.

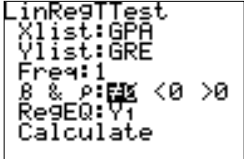


Figure 13.21: LinRegTTest input screen.

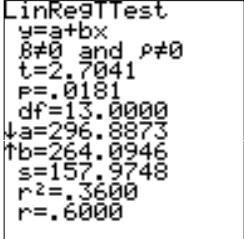


Figure 13.22: LinRegTTest results.

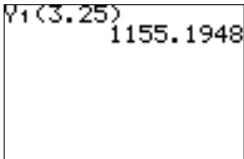


Figure 13.23: Predicted GRE score for $\text{GPA} = 3.25$.

predicts a student with GPA 3.25 will score approximately 1155 on the graduate record exam. \square

The regression equation may be used to make predictions for $\mu_{y|x_0}$, the mean value of Y for a given value of x , and for y_{x_0} , a random value of Y for a given value of x . The best point estimate for both $\mu_{y|x_0}$ and y_{x_0} is \hat{y} . The calculator program REGINTS may be used to construct a confidence interval for a mean response, a prediction interval for a random value, and to sketch confidence interval bands and prediction belts.

```
LinReg
y=a+bx
a=7.1608
b=-.0373
r2=.3167
r=-.5628
```

Figure 13.24: LinReg(a+bx) results.

```
XLIST: LSTREN
YLIST: LFINE
```

Figure 13.25: REGINTS data input.

```
PROCEDURE
1:ACT MEAN RESP
2:CI RAND VALUE
3:CONF BAND
4:PRED BELT
```

Figure 13.26: REGINTS procedure menu.

```
C-LEVEL: .99
X: 80
```

Figure 13.27: Confidence level and x value

```
CI ENDPOINTS FOR
MEAN RESPONSE
4.0174
4.3362
Done
```

Figure 13.28: Confidence interval for a mean response.

Example: A recent study reported on the relationship between the *strength* and *fineness* of cotton fibers. Sixteen random samples were selected and the strength and fineness for each fiber were recorded. The data is given in the table below.

Strength (x)	76	69	71	76	83	72	78	74
Fineness (y)	4.4	4.6	4.6	4.1	4.0	4.1	4.9	4.8
Strength (x)	80	82	90	81	78	80	81	78
Fineness (y)	4.2	4.4	3.8	4.1	3.8	4.2	3.8	4.2

- (E1) Calculate the regression equation using strength as the independent variable.
- (E2) Find a 99% confidence interval for the mean measurement of fineness for fibers with a strength of 80.
- (E3) Find a 99% prediction interval for a random value of fineness for fibers with a strength of 75.
- (E4) Sketch a scatter diagram, the regression line, and a 95% confidence band on the same coordinate axes.

Calculator Solution:

- (S1) Enter the strength data in the list STREN and the fineness data in the list FINE.
- (S2) Execute LinReg(a+bx) using STREN as the independent variable list and FINE as the dependent variable list. The LinReg(a+bx) results are displayed on the Home Screen (Figure 13.24). The regression equation is $\hat{y} = 7.1608 - .0373x$.
- (S3) Execute the program REGINTS. Input the independent variable list and the dependent variable list (Figure 13.25). Select confidence interval for a mean response from the program procedure menu (Figure 13.26).
- (S4) Enter the confidence level and a value for x (Figure 13.27). The confidence interval is displayed on the Home Screen (Figure 13.28). (4.0174, 4.3362) is a 99% confidence interval for the mean fineness when the strength is 80.

- (S5) Execute the program `REGINTS`. This time select a prediction interval for a random value from the procedure menu. Enter the confidence level and the value for x . The prediction interval is displayed on the Home Screen (Figure 13.29). (3.7436, 4.9830) is a 99% prediction interval for a random fineness when the strength is 75.
- (S6) Execute the program `REGINTS`. Select confidence band from the procedure menu and enter a confidence level. The scatter diagram, graph of the regression equation, and the confidence bands are displayed (Figure 13.30).

Note: The program uses `ZoomStat` to simultaneously display the scatter diagram, regression line, and confidence band (or prediction belt). The equations and lists are stored so that you may adjust the `WINDOW` settings and change the graph style(s) if desired, and redraw the graphs.

◇ □

```

CI ENDPOINTS FOR
RANDOM VALUE
3.7436
4.9830
Done

```

Figure 13.29: Prediction interval for a random value.

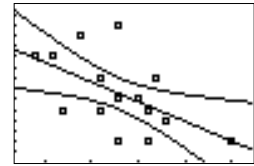


Figure 13.30: Scatter diagram, regression line, and confidence bands.

Notes:

Chapter 14

Elements of Nonparametric Statistics

Most of the statistical tests described in this manual require certain assumptions; for example, the population is normal, or the population variances are equal. **Nonparametric tests** are important alternatives if few assumptions can be made about the population(s) being studied. These **distribution-free** procedures are based on elementary probability theory, are extremely versatile, and are easy to use once a table of critical values is developed for a particular application.

The **sign test** is a widely used nonparametric test that uses only plus and minus signs. Three applications of the sign test are:

- (A1) A confidence interval for the median of a single population.
- (A2) A hypothesis test concerning the value of the median for a single population.
- (A3) A hypothesis test concerning the median difference (paired difference) for two dependent populations, often *before* and *after* treatment measurements on the same individual or object.

The only assumption is that the n random observations forming the sample are selected independently and the population is continuous in the vicinity of the median, \tilde{u} .

The test statistic is the number of plus signs (for example, the number of observations greater than the hypothesized median). If the null hypothesis is true, there will be an equal chance for plus and minus signs, and the test statistic will have a binomial distribution with $p = .5$. If the number of plus signs is significantly large or significantly small then there is evidence of a treatment effect.

The sign test:

$$H_0: \tilde{\mu} = \tilde{\mu}_0$$

$$H_a: \tilde{\mu} > \tilde{\mu}_0, \quad \tilde{\mu} < \tilde{\mu}_0, \quad \tilde{\mu} \neq \tilde{\mu}_0$$

TS: X = the number of plus signs

Under the null hypothesis, X has a binomial distribution with parameters n and $p = .5$.

$$\text{CR: } X \geq c_1, \quad X \leq c_2, \quad X \geq c \text{ or } X \leq n - c$$

The critical values c_1 , c_2 , and c are obtained from the binomial distribution with parameters n and $p = .5$ so that the probability of a Type I error is as close as possible to the desired significance level without exceeding it.

Sample values equal to $\tilde{\mu}_0$ are excluded from the analysis and the sample size is reduced accordingly.

If $n \geq 10$, the distribution of X is approximately normal with mean $.5n$ and variance $.25n$. In this case, a normal approximation to the binomial distribution may be used to obtain a test statistic based on a standard normal random variable.

$$\text{TS: } z^* = \frac{(X \pm .5) - .5n}{.5\sqrt{n}}$$

The continuity correction, $\pm .5$, is used and depends upon the alternative hypothesis.

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

The calculator program **SIGNTEST** will compute the position numbers for a confidence interval, construct a confidence interval for the median of a single population, and conduct a hypothesis test concerning the median for a single population or the median difference for two dependent populations. The program computes both test statistics and returns the observed significance level in each case.

Example: A recent article suggested that university administrator's pay increased by 5.4% last year. A national teacher's union argued the pay increase was much more. In order to investigate the claim made in the article, a random sample of university administrator salary increases was obtained. The percentage increases are given in the table below.

1.6	11.4	4.2	8.7	4.5	6.4	5.9	5.7	4.5	6.6	4.4	4.1
7.6	5.5	10.6	7.8	6.8	3.0	3.5	7.9	8.0	4.3	2.4	7.0

Is there any evidence to suggest the median pay increase is greater than 5.4%? Use $\alpha = .05$.

Calculator Solution:

(S1) The 24 observations were obtained randomly and the population is assumed to be continuous in the vicinity of the median. There are no other assumptions about the population. The parameter of interest is the median. A sign test is appropriate.

(S2) The four parts of the hypothesis test are:

$$H_0: \tilde{\mu} = 5.4 (\leq)$$

$$H_a: \tilde{\mu} > 5.4$$

TS: X = the number of plus signs (the number of observations greater than 5.4).

$$\text{For large } n: z^* = \frac{(X \pm .5) - .5n}{.5\sqrt{n}}$$

$$\text{CR: } X \geq 17 \quad (\alpha \approx .0320)$$

$$\text{Using } z^*: z^* \geq z(.05) = 1.6449$$

(S3) Enter the salary increase data in the list PAY. Execute the program SIGNTEST.

(S4) Select hypothesis test from the procedure prompt (Figure 14.1) and data contained in one list from the input prompt (Figure 14.2). Enter the list containing the salary data and the hypothesized median (Figure 14.3), and select the alternative hypothesis (Figure 14.4).

(S5) The hypothesis test results are displayed on the Home Screen (Figure 14.5). Both test statistics (X , and the large-sample z^*) and corresponding P values are given.

(S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $x = 14$ and $z^* = .6124$, does not lie in the critical region (equivalently, $P = .2706$; $P = .2701$). There is no evidence to suggest the median salary increase is greater than 5.4%. \square

The Mann–Whitney U test is a nonparametric alternative to the t test for the difference between two population means. It is designed to help determine if two independent samples come from the same populations. Assume the two independent random samples are independent within each sample as well as between samples and are from continuous distributions.

The hypothesis test is stated in terms of population medians and is based on the sum of ranks. The data from the two independent samples is combined and ranked. If the two samples come from the same population, then the ranks should be evenly distributed. Otherwise one of the samples will have more of the lower ranks and thus a smaller sum of ranks. If both

```
PROCEDURE
1:CI-POS NUMBS
2:CI-DATA
3:HYP TEST
```

Figure 14.1: Sign test procedure.

```
INPUT
1:STATS
2:DATA: 1 LIST
3:DATA: 2 LISTS
```

Figure 14.2: Hypothesis test input.

```
DATA: LPAY
MED0 = 5.4
```

Figure 14.3: List containing the data and $\tilde{\mu}_0$.

```
ALT HYP
1:MED > MED0
2:MED < MED0
3:MED ≠ MED0
```

Figure 14.4: Alternative hypothesis.

```
H0: MED = 5.4000
HA: MED > 5.4000
TS(X) = 14.0000
P = .2706
TS(Z) = .6124
P = .2701
```

Figure 14.5: Hypothesis test results.

samples are eight or larger, then the test may be based on the standard normal distribution. The calculator program MANNWHIT may be used to conduct this test.

The Mann–Whitney U test:

Let X_1, X_2, \dots, X_m and Y_1, Y_2, \dots, Y_n , represent the independent random samples.

$$H_0: \tilde{\mu}_1 - \tilde{\mu}_2 = \Delta_0$$

$$H_a: \tilde{\mu}_1 - \tilde{\mu}_2 > \Delta_0, \quad \tilde{\mu}_1 - \tilde{\mu}_2 < \Delta_0, \quad \tilde{\mu}_1 - \tilde{\mu}_2 \neq \Delta_0$$

Subtract Δ_0 from each observation in sample 1. Combine the $(X_i - \Delta_0)$'s and the Y_j 's into one sample and rank all of the observations. Equal observations are assigned the mean rank for their positions.

$$\text{TS: } U^* = \min\{U_1, U_2\}$$

$$U_1 = mn + \frac{n(n+1)}{2} - R_2, \quad U_2 = mn + \frac{m(m+1)}{2} - R_1$$

R_1 is the sum of the ranks of $(X_i - \Delta_0)$ in the combined sample and R_2 is the sum of the ranks of Y_j in the combined sample.

$$\text{CR: } U^* \leq c$$

c is a critical value (from a table) for the Mann–Whitney U test so that the probability of a Type I error is as close as possible to the desired significance level without exceeding it.

When both m and n are greater than 8, U has approximately a normal distribution with

$$\mu_U = \frac{mn}{2} \quad \text{and} \quad \sigma_U^2 = \frac{mn(m+n+1)}{12}$$

In this case the Mann–Whitney U test may be based on a standard normal distribution.

$$\text{TS: } z^* = \frac{U^* - \mu_U}{\sigma_U}$$

$$\text{CR: } z^* \leq -z(\alpha) \quad \text{or} \quad z^* \leq -z(\alpha/2) \quad (\text{if two-tailed test})$$

Example: The data given in the table below represents the number of hours two different cellular phone batteries worked before a recharge was necessary.

Battery A	44	42	44	44	42	49	53	58
Battery B	51	52	45	47	49	41	48	37
Battery A	41	49	47	45				
Battery B	40	49	43	53	44	55	35	

Is there any evidence to suggest battery type B lasts longer than battery type A. Use $\alpha = .05$.

Calculator Solution:

(S1) Assume the two samples are independent and the battery life distributions are continuous. In order to compare the distributions a Mann–Whitney U test is appropriate.

(S2) The four parts of the hypothesis test are:

$$H_0: \tilde{\mu}_1 - \tilde{\mu}_2 = 0 (\geq)$$

The distribution of battery life is the same for each type.

$$H_a: \tilde{\mu}_1 - \tilde{\mu}_2 < 0$$

The distributions are not the same. Battery type B lasts longer than battery type A.

$$TS: U^* = \min\{U_1, U_2\}$$

Both sample sizes are greater than 8. U has approximately a normal distribution.

$$z^* = \frac{U^* - \mu_U}{\sigma_U}$$

$$CR: U^* \leq 55$$

Using z^* : $z^* \leq -z(.05) = -1.6449$

(S3) Enter the times for battery A in the list `BATTA` and the times for battery B in the list `BATTB`. Execute the program `MANNWHIT`.

(S4) Enter the name of each list containing the data and the value for Δ_0 (Figure 14.6). Select the alternative hypothesis (Figure 14.7).

(S5) The hypothesis test results are displayed on the Home Screen (Figure 14.7). The values for U^* and z^* are given along with the p -value and the sample sizes.

(S6) Decision: do not reject H_0 .

Conclusion: The value of the test statistic, $U^* = 77.5$ and $z^* = -.6099$, does not lie in the critical region (equivalently, $P = .2710 > .05$). There is no evidence to suggest the distributions are different, that battery B lasts longer than battery A. \square

```

XLIST: LBATTA
YLIST: LBATTB
NULL HYPOTHESIS
D0 = 0
  
```

Figure 14.6: Lists containing data and value for Δ_0 .

```

ALT HYP
1: U1-U2 > D0
2: U1-U2 < D0
3: U1-U2 = D0
  
```

Figure 14.7: Alternative hypothesis.

```

H0: D0 = 0.0000
HA: D0 < 0.0000
U = 77.5000
n1 = 12.0000
n2 = 15.0000
Z = -.6099
P = .2710
  
```

Figure 14.8: Hypothesis test results.

The runs test is used to test the randomness of data. Observations are divided into two categories according to a certain characteristic. A run is a sequence of data that possesses a common characteristic. The test statistic is V^* , the number of runs observed. An approximate test procedure is based on the standard normal distribution. The program `RUNSTEST` will conduct this approximate test procedure given the number of runs.

The runs test:

H_0 : The sequence is random.

H_a : The sequence is not random.

TS: V^* = the number of runs.

CR: $V^* \leq c_1$ or $V^* \geq c_2$

c_1 and c_2 are critical values (from a table) for the runs test so that the probability of a Type I error is as close as possible to the desired significance level without exceeding it.

Let n_1 be the number of observations with characteristic 1, and let n_2 be the number of observations with characteristic 2. When both n_1 and n_2 are greater than 20, V has approximately a normal distribution with

$$\mu_V = \frac{2n_1n_2}{n_1 + n_2} + 1 \quad \text{and} \quad \sigma_V^2 = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}$$

In this case the runs test may be based on the standard normal distribution.

$$\text{TS: } z^* = \frac{V^* - \mu_V}{\sigma_V}$$

$$\text{CR: } |z^*| \geq z(\alpha/2)$$

Example: The letters in the following sequence represent the direction each car turned after exiting at a certain ramp on the New Jersey Turnpike (L - left, R - right).

L L R R R R R R L L L R R R L L R R
 R R L L L L R R L L L R L R L L R R L
 L R R R R R L L L L L R R R R R L L

Test the null hypothesis that the sequence is random with regards to direction. Use $\alpha = .05$.

Calculator Solution:

(S1) A runs test is appropriate in order to test the sequence for randomness. The four parts of the hypothesis test are:

H_0 : The sequence of directions is random.

H_a : The sequence of directions is not random.

TS: V^* = the number of runs.

The number of observations with each characteristic is greater than 20. V has approximately a normal distribution.

$$z^* = \frac{V^* - \mu_V}{\sigma_V}$$

$$\text{CR: } |z^*| \geq z(\alpha/2) = z(.025) = 1.96$$

- (S2) Execute the program RUNSTEST. Enter the number of observations with each characteristic (the number of L's and the number of R's) and the number of runs (Figure 14.9).
- (S3) The hypothesis test results are displayed on the Home Screen (Figure 14.10).
- (S4) Decision: reject H_0 .

Conclusion: The value of the test statistic, $z^* = -3.0782$, lies in the critical region (equivalently, $P = .0021 < .05$). There is evidence to suggest the direction cars turn after exiting the New Jersey Turnpike at this exit is not random. \square

The sample linear correlation coefficient is a measure of the strength of the linear association between two continuous variables. **Spearman's rank correlation coefficient** is a measure of the correlation between paired rankings.

Suppose there are n pairs of observations from a random sample and the variables are ordinal or numerical. If necessary, convert the data to ranks, and rank each sample separately. Equal observations are assigned the mean rank for their position. Spearman's rank correlation coefficient is a measure of the linear association between the ranks and is defined by

$$r_s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}$$

where d_i is the difference in the paired rankings. The formula is valid only if there are no ties. However, the approximation is good if the number of ties is small in comparison to n .

As n increases the distribution of r_s approaches a normal distribution. The calculator program **SPEARMAN** computes r_s and conducts a hypothesis test based on the standard normal distribution. Tables are available for critical values of Spearman's rank correlation coefficient.

Hypothesis test about a population correlation coefficient between ranks based on Spearman's rank correlation coefficient:

$H_0: \rho = 0$ (no correlation between the ranks in the population)

$H_a: \rho > 0, \quad \rho < 0, \quad \rho \neq 0$

TS: r_s^*

```
n1 = 27
n2 = 33
V = 19
```

Figure 14.9: Number of observations with each characteristic and number of runs.

```
H0: RANDOM
HA: NOT RANDOM
Z = -3.0782
P = .0021
```

Figure 14.10: Hypothesis test results.

$$\text{CR: } r_s^* \geq c_1, \quad r_s^* \leq c_2, \quad |r_s^*| \geq c$$

c_1 , c_2 , and c are critical values from a table.

As n increases, r_s has approximately a normal distribution. In this case the hypothesis test may be based on the standard normal distribution.

$$\text{TS: } z^* = r_s \sqrt{n-1}$$

$$\text{CR: } z^* \geq z(\alpha), \quad z^* \leq -z(\alpha), \quad |z^*| \geq z(\alpha/2)$$

Example: A researcher believes a certain toxic chemical accumulates in body tissues with age and may eventually cause heart disease. Twelve subjects were selected at random. Their age and the chemical concentration (in parts per million) in tissue samples is given in the table below.

Age	82	83	64	53	47	50
Chem Con	170	40	64	5	15	5
Age	70	62	34	27	75	28
Chem Con	48	34	3	7	50	10

Is there any evidence to suggest the chemical concentration in tissue samples increases with age? $\alpha = .01$.

Calculator Solution:

(S1) Assume the 12 ordered pairs of data form a random sample and both variables are continuous. Spearman's rank correlation coefficient is an appropriate measure of the linear association between age and chemical concentration ranks.

(S2) The four parts of the hypothesis test are:

$$H_0: \rho = 0 (\leq)$$

There is no correlation between the ranks in the population.

$$H_a: \rho > 0$$

There is a positive rank correlation in the population.

$$\text{TS: } r_s^*$$

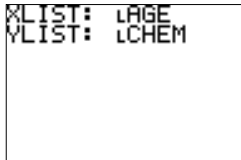
The approximate test statistic based on the standard normal distribution is given by: $z^* = r_s \sqrt{n-1}$

$$\text{CR: } r_s^* \geq .703$$

$$\text{Using } z^*: z^* \geq z(\alpha) = z(.01) = 2.3263$$

(S3) Enter the age data in the list **AGE** and the chemical concentration data in the list **CHEM**. Execute the program **SPEARMAN**.

(S4) Enter the names of the lists containing the data (Figure 14.11),



```

XLIST: LAGE
YLIST: LCHEM
  
```

Figure 14.11: Lists containing the data.

select the type of data (Figure 14.12), and choose the alternative hypothesis (Figure 14.13).

(S5) The hypothesis test results are displayed on the Home Screen (Figure 14.14). The value of Spearman's rank correlation coefficient is included in the output and labeled as **R**. Here, $r_s = .7535$.

(S6) Decision: reject H_0 .

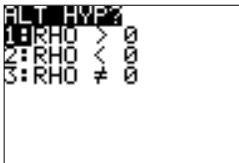
Conclusion: The value of the test statistic, $r_s^* = .7535$ and $z^* = 2.4991$, lies in the rejection region (equivalently, $P = .0062 < .01$). There is evidence to suggest that older people tend to have higher levels of chemical concentration in their tissues. \square



```

DATA?
1: UNRANKED
2: RANKS
  
```

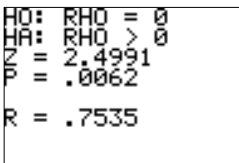
Figure 14.12: Type of data.



```

ALT HYP?
1: RHO > 0
2: RHO < 0
3: RHO ≠ 0
  
```

Figure 14.13: Alternative hypothesis.



```

HO: RHO = 0
HA: RHO > 0
Z = 2.4991
P = .0062

R = .7535
  
```

Figure 14.14: Hypothesis test results.

Notes:

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Calculator Programs and Lists

The table below contains a list of the TI-83 Plus calculator programs illustrated in this supplement, a brief description of each program, and the page on which the program is first presented.

Program	Description	Page
BAYES	Applies Bayes' Rule to a collection of conditional and unconditional probabilities.	38
CENTRAL	Computes the midrange and mode for a data set.	18
CHEBY	Applies Chebyshev's theorem to a sample or population, summary statistics, or observations.	25
CHISQINT	Constructs a confidence interval for a population variance, σ^2 .	68
CHISQTST	Conducts a hypothesis test about a population variance, σ^2 .	69
CIRCLE	Displays a circle graph or pie diagram.	9
CROSSTAB	Summarizes data in a cross-tabulation or contingency table.	27
DISCRV	Computes the mean, variance, and standard deviation for a discrete random variable.	41
DOTPLOT	Displays a dotplot.	13
FREQDIST	Constructs a frequency distribution for a data set.	13
GOODNESS	Conducts a goodness-of-fit test to determine how well an observed set of frequencies fits an expected set of frequencies.	84
GROUPBAR	Displays side-by-side bar graphs.	29
GROUPED	Computes (estimates of) the mean, variance, and standard deviation for data presented in the form of a grouped frequency distribution.	20
PERCENT	Displays a Pareto diagram for numbers and types of defects.	12
POSITION	Computes quartiles, percentiles, midquartile, and interquartile range for a data set.	21
RANDOM	Constructs a simple random sample with or without replacement.	53

The table below contains the TI-83 Plus calculator lists and groups (of lists) used in examples presented in this supplement that are available at the web site listed in the Preface. Calculator group names are indicated with an asterisk in front of the name.

List	Description	Page
LAPT	Aptitude test scores for applicants for a certain civil service position.	24
LBLOOD	Hemoglobin test results from diabetics.	14
LBOXES	Weight of randomly selected cereal boxes.	61
*CHICK: LFREQS, LGAIN	Data from an experiment to judge the value of a new diet for chickens. Frequency of occurrence of each observation and weight gain.	20
LCLASS	Number of students registered for introductory statistics class sections.	84
*COLLS: LGEND, LMAJOR	College students classified according to gender and major.	27
*COLWM: LTALL, LWEIGH	Height and weight of eight college women.	33
LDAYS	Days in a standard school year for several different countries.	17
LDEFEC	Final daily inspection defect report for an assembly line at a local manufacturer.	12
*FASTFOOD: LCALOR, LFAT	Calories and fat content in some popular fast food items.	30
LIGNIT	Ignition time data for certain fabrics.	21
*LONGDIST: LCOMPA, LCOMPB, LCOMPC	Long distance telephone charges for randomly selected phone numbers using one of three different carriers: A, B, C.	29
LLUNCH	Daily amount spent for lunch by employees at a small business.	19
LMILES	Miles per gallon for a specially designed high mileage, two-seat automobile.	69
LMINS	Exercise capacity (measured in minutes) for police academy recruits.	18
LNUMB	Distribution of automobiles in the faculty parking lot on a recent Monday morning.	11
*NUNS: LAGE, LCOUNT	Roman Catholic nun survey results. Class marks for ages and frequency for each class interval.	16
LOCTAN	Gasoline octane ratings from a supplier's pipeline.	64

List	Description	Page
LOPERS	Number of cases of each type of operation performed at General Hospital last year.	10
LPARTS	Length of parts produced by a certain machine after an adjustment.	58
*PECLASS: LPUSH, LSIT	Fitness scores from a physical education class: push-ups and sit-ups.	32
*SALTFREE: LBEFOR, LAFTER	Before and after diastolic blood pressure for people following a salt-free diet for two weeks.	72
LSCORE	A random sample of exam scores from a large introductory statistics class.	13
*SYSTOLIC: LSBPM, LSBPW	Systolic blood pressure for men and women.	80
LTIMES	Manual dexterity times required to complete a certain task.	23
*VACATION: LSTUDY, LUNIT	Vacation home market study: percentage of people who preferred each unit size and survey results.	86