Applied Statistics for the Behavioral Sciences

Chapter 8
One-sample designs
Hypothesis testing/effect size

Chapter Outline

- Hypothesis testing
  - null & alternative hypotheses
  - alpha (α), significance level, rejection region, & critical values
- One-sample t test
- Converting r to t
- Interpretation of *p*
- Decisions about the null hypothesis
- One tailed and two tailed tests
- Effect size index

Hypothesis testing

- belief about the state of affairs in nature
- how some group compares with another
- one sample case
  - measure a single group
  - compare their scores with standard/population parameter.
- make decision - about true state of nature
- establish difference between data and some population parameter

Hypothesis testing (cont.)

- set up conditions so we are predicting that the values we observe will be different than some parameter
- if wrong, data will be within limits expected if no difference existed in nature
- if right about a difference existing, data will not be within range expected to occur if no difference.
- testing a hypothesis about equality.
- Research hypothesis is that the equality condition is not true
- end result: decision about null hypothesis.

An example

- During recent year, the mean SAT score (math + verbal) was 896
- summary statistics for a small high school
- is this school's performance equal to the national norm?
- The Data:
- $\Sigma X = 24{,}206$ $\Sigma X^2 = 22{,}716{,}411$
- $N = 26$ $(\Sigma X)^2 = 5.8593$ E8 (or 585,930,000)
- $\bar{X} = 931$ $\hat{s} = 85$
- sem = 16.670

Possible relationships between our high school and pop value

- $\mu_1 = \mu_0$
- $\mu_1 < \mu_0$
- $\mu_1 > \mu_0$
- observed mean of 931 is larger than population parameter of 896
- is this deviation one that is reasonably likely due to chance fluctuation alone?

Two hypotheses

- Set up two hypotheses that cover all possible values of the parameter
- first is the hypothesis of equality, our data came from a population with a mean of 896
- second hypothesis is that data did not come from a population with a mean of 896.

Which is correct?

- tentatively assume equality hypothesis correct
- calculate the probability of the statistic observed in our sample
- statistic is the t statistic
- null hypothesis=our hypothesis of equality
  - the hs we have data for no different than the national average of 896
  - $H_0 : \mu = 896$

Alternative hypotheses

- potentially infinite number of alternative hypotheses
- alternatives framed consistently with research hypothesis
- interested in establishing the likelihood that our hs mean came from a population with a mean of 896
- $H_1 : \mu \neq 896$

Making the decision

- large difference between our observed data and hypothesized value makes us unlikely to accept the assumption that deviation arose by chance alone.
- difference of 35 points
- compare difference with sampling distribution of t and determine whether it is a difference that would be expected

Example particulars

- null hypothesis= there is not a difference between our HS mean SAT score and the national average of 896
- can be thought of as the hypothesis the researcher hopes to disprove or "nullify"
- Rejection of the null tells us something, failure to reject leaves us with the null as one of many possible hypotheses.

Alpha

- Where do we set our cut off for what constitutes a "chance deviation" vs. a statistically significant difference?
- setting alpha will generally be simply using the "default" value we have discussed, .05, this represents our "break point"
- we reject the null for any result with a $p <= .05$ and retain the null for any result $> .05$

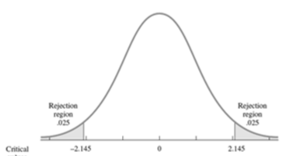
Significance level

- The actual probability of observing a difference as large or larger than the one found in the data is the significance level
- SPSS provides actual significance levels
- even when we calculate t by hand we can see in the table whether our t statistic exceeds smaller probability levels or not.

Rejection region

- alpha of .05, 2.5% of area in each tail
- alpha of .01, 0.5% of area in each tail.
- Critical values
  - value that must be exceeded by the t statistic for a statistically significant difference
  - to reject the null at various values of alpha
  - critical value itself is part of the rejection region

Fig 9.2 pg 184
t dist w/14 df, for α=.05



One-sample t test

- can simplify the process we just went through and formalize it into the one-sample t test formula
- with N-1 df

$$t = \frac{\overline{X} - \mu_0}{s_{\overline{x}}}$$

Converting r to t

- test whether a correlation coefficient differs significantly from zero
- treat sample correlation as a sample mean and the population correlation coefficient (rho) as the "test value" in this case equal to zero

$$t = \frac{r - \rho}{s_r} \qquad s_r = \sqrt{\frac{(1 - r^2)}{N - 2}}$$

Converting r to t (cont.)

- use the standard error of the sample correlation coefficient
- sampling distributions for population correlation coefficients other than 0 are not t distributions

$$t = (r)\sqrt{\frac{N - 2}{1 - r^2}}$$

Interpretation of *p*

- p value associated with our statistical test represents the probability of a result as large or larger if null true
- Decisions about the null hypothesis
  - accepting or rejecting

		True state of nature	
		H_0 true	**H_0 false**
Decision based on data	**Do not reject H_0**	**Correct decision**	**Type 2 error ($p=\beta$)**
	Reject H_0	**Type 1 error ($p=\alpha$)**	**Correct decision**

One tailed and two tailed tests

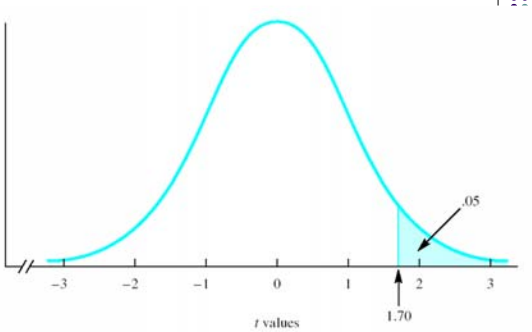


Figure 9.5 (p. 191): A one-tailed test of significance, with α = .05, *df* = 30. The critical value is 1.70.
What would the critical value be for 2 tailed test?

Effect size index

- gives us a yardstick for comparing the magnitude of differences across studies.
- the effect size index for a one sample t is d and obtained from the following:

$$d = \frac{\overline{X} - \mu_0}{\sigma}$$

Review guidelines, material re: *d* from Ch 5