Basic Statistics

Chapter 9
Two sample tests

Chapter Outline

- Simple experiments
- Two-sample hypothesis testing
- Distinguishing between independent and correlated samples designs
- Calculation of t
  - Independent samples
  - Correlated samples
- Assumptions that accompany t
- Effect size
- Power
- Practical vs. statistical significance

Homework problems

- Homework problems should be done both by hand and in SPSS until clearly understanding and able to run in SPSS w/o difficulty
- Problems on handout (from study guide) we will do in class both ways, work on your own ASAP

Simple experiment steps (p 195)

- *Identify population*
- *RANDOMLY assign to groups*
- *Administer treatment (IV) [treatment A]*
- *Measure performance/behavior (DV) [task Q]*
- *Calculate means for groups (Descr. Stats.)*
- *Calculate effect size index* $\overline{X}_e$ & $\overline{X}_c$
- *Compare group means (Inferential statistics)*
- *Write a conclusion–in terms of original question*

Experiments

- allow causal statements
- by randomly assigning to groups we've ruled out alternatives
- left with our intervention

Two-sample hypothesis testing (p 196-197)

- ***two possibilities null and alternative hypothesis***
  H_0 : no effect, mean score for those receiving the treatment is equal to the mean for those not receiving [text – A vs. noA]
  $H_0 : \mu_{treat} - \mu_{no\ treat} = 0$ or $H_0 : \mu_{treat} = \mu_{no\ treat}$
  $H_1 : \mu_{treat} \neq \mu_{no\ treat}$ ***(2 tailed) or either***
  $H_1 : \mu_{treat} > \mu_{no\ treat}$ or $\mu_{treat} < \mu_{no\ treat}$ ***(1 tailed)***

Two-sample hypothesis testing

- *assume equality hypothesis correct If null true: samples vary only by extent of sampling error*
- *select alpha level (almost always will be .05)*
- *choose an inferential statistic (for now just need to decide which t, independent or correlated samples)*
- *calculate test statistic from sample data*

Two-sample hypothesis testing

- *compare the test statistic with the critical value from the statistic's sampling distribution*
  - *if the t value obtained from the data exceeds t critical (or probability < alpha) we can reject the null*
  - *if probability is greater than alpha (or statistic doesn't exceed critical value) retain the null.*
- *formulate a conclusion/interpretation – again in the terms of the original question*

Independent vs. correlated t tests

- basic idea is taking a difference observed, divide by standard error of the difference giving a t statistic
- Three types of correlated designs:
  - *Natural pairs*
  - *Matched pairs*
  - *Repeated measures*
- Independent samples
  - no way to pair observations

Calculation of t

- Independent samples design
  - df = $N_1 + N_2 - 2$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}}$$

Standard error of difference

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{s_{\bar{X}_1}^{\ 2} + s_{\bar{X}_2}^{\ 2}}$$

- If $N_1 = N_2$

$$= \sqrt{\left(\frac{\hat{s}_1}{\sqrt{N_1}}\right)^2 + \left(\frac{\hat{s}_2}{\sqrt{N_2}}\right)^2}$$

$$= \sqrt{\frac{\Sigma X_1^{\ 2} - \frac{(\Sigma X_1)^2}{N_1} + \Sigma X_2^{\ 2} - \frac{(\Sigma X_2)^2}{N_2}}{N_1(N_2 - 1)}}$$

- If N_1 ne N_2

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\Sigma X_1^{\ 2} - \frac{(\Sigma X_1)^2}{N_1} + \Sigma X_2^{\ 2} - \frac{(\Sigma X_2)^2}{N_2}}{N_1 + N_2 - 2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}$$

Independent samples T

- Lazarus data
- Do by hand
- Later in SPSS, answers should match

Detached	Involved
23	31
21	27
19	24
15	23
14	21
12	14
10	

Effect size

- Cohen's d statistic, same as one-sample case
- Interpretation is identical
- Tells us how much of an effect the independent variable had

Effect size index

- Independent samples $d = \frac{\overline{X}_1 - \overline{X}_2}{\hat{s}}$
- if N1 = N2, $\hat{s} = \sqrt{N_1 / 2}(s_{\bar{x}_1 - \bar{x}_2})$
- If N1 ne N2, $\hat{s} = \sqrt{\frac{\hat{s}_1^{\,2}(df_1) + \hat{s}_2^{\,2}(df_2)}{df_1 + df_2}}$

Correlated samples t-test

- df = N - 1 where N = number of pairs of scores

$$t = \frac{\overline{X} - \overline{Y}}{s_{\overline{D}}}$$

- Formula illustrates effect r has

$$s_{\overline{D}} = \sqrt{s_{\overline{X}}^{\;2} + s_{\overline{Y}}^{\;2} - 2r_{XY}(s_{\overline{X}})(s_{\overline{Y}})}$$

Direct difference method

- does not require calculating r first

$$t = \frac{\bar{X} - \bar{Y}}{s_{\bar{D}}} = \frac{\bar{X} - \bar{Y}}{\hat{s}_D / \sqrt{N}} \qquad \hat{s}_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{N}}{N - 1}}$$

- D = X - Y
- N = number of pairs of scores

Correlated Samples t – example

- Two cats from each litter randomly assigned to each group
- Amount of alcohol laced milk they drank was measured

Littermates	No Experimental Neurosis	Experimental Neurosis
1	63	88
2	59	90
3	52	74
4	51	78
5	46	78
6	44	61
7	38	54

- Perform a t-test, an effect size index and calculate 99% confidence interval.

Effect size index

- For correlated samples, same general formula:

$$d = \frac{\bar{X} - \bar{Y}}{\hat{s}}$$

- Difference is in $\hat{s} = \sqrt{N}\,(s_{\bar{D}})$

Assumptions of t

- dependent variable normally distributed
- dv variance = across groups
- samples randomly selected
- t test is robust to violations of assumptions

Final issues

- Power
  - only way we are able to control power is via sample size
  - comparing groups want to be certain we're using a relatively sensitive measure that will reveal group differences of a reasonable magnitude
- Practical vs. statistical significance
  - Finding statistical significance tells us nothing about the practical or clinical significance of a research finding – effect size plays a role here