Applied Statistics for the Behavioral Sciences

Chapter 8

Samples, Sampling Distributions, Confidence Intervals (and a first look at hypothesis testing and the t distribution)

1

Outline

- Sampling distributions (sampling distribution of the mean)
- Central limit theorem
- Standard error of the mean
- The z test
- Confidence intervals
- The t distribution
- When to use the normal or t distribution

2

Determining Chance Events

- ESP claims
- Turns over pieces of paper, labeled A and B in correct order
- Convinced?
- A B C?
- How many do you need to see?
- Chance variation vs. significantly different
- Descriptive vs. inferential statistics

3

Descriptive vs. inferential

- Descriptive statistics: simply describing a collection of data
- Inferential statistics: making inferences beyond the data collected
- Describing samples can be interesting
- Usually interested in estimating population parameters
- Sample mean is unbiased estimator of the population mean
- don't know how close we are to true population mean

4

Sampling distributions – basic idea

- Draw all possible random samples of size n from a population of N
- Calculate the mean of each sample
- Mean of a sampling distribution of means will equal the population mean
- The sampling distribution will be normally distributed with a mean of $\mu_{\bar{x}}$ and s.d. of $\frac{\sigma}{\sqrt{n}}$

5

Central limit theorem

- In a population with a mean μ and a standard deviation σ
- distribution of sample means approaches a normal distribution with a mean $\mu_{\bar{x}}$ and standard deviation of $\frac{\sigma}{\sqrt{n}}$
- If the sample size is sufficiently large, the sampling distribution of means will be approximately normal regardless of the shape of the population distribution
- What is sufficiently large?

6

Normal sampling distribution

- A normal distribution, characteristics all apply
- 68% of all sample means will be within ± 1 s.d. from $\mu_{\bar{x}}$
- 95% between ± 1.96 s.d., total of 5% below -1.96 and above 1.96, 2.5% of the area in each tail
- Standard error of the mean = s.d. of the sampling distribution of the mean
- $\sigma_{\bar{x}}$ (or SEM) = $\dfrac{\sigma}{\sqrt{n}}$

Standard Error of the Mean

- This lets us calculate the probability of various sample means given a certain population mean
- What happens to the SEM as N increases?
- Hours worked per week at part time jobs by 100 college students
- $\bar{X}$ = 11.7 and we know that the population s.d. σ = 4.0

$$\sigma_{\bar{x}} = \frac{4}{\sqrt{100}} = .4$$

- With a sample mean of 11.7 and SEM = .4 "is it possible" for the population mean, μ , to be 12?

Sample mean probabilities

- We need a new kind of z score to answer this question
- x = observation
- $\bar{x}$ = sample mean
- s = sample standard deviation
- Replace variables as follows:
- X becomes the sample mean
- $\bar{x}$ becomes the mean of the sampling distribution
- s is replaced with the s.d. of the sampling distribution of the mean (SEM)

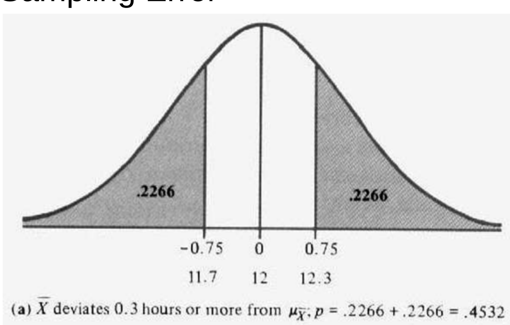
$$z = \frac{x - \bar{x}}{s}$$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

Sample mean probabilities

- (11.7-12)/0.4 = -.3/.4 = -0.75
- By sampling error alone, sample means above and below the mean are equally likely
- interested in the probability that a mean could vary this much or more in either direction
- Area under the normal curve beyond a z of .75 in both directions
- Area = 22.66%, doubling = 45.32%
- What would we say about the possibility of a sample mean of 11.7 if true pop. mean is 12?
- Can view graphically.

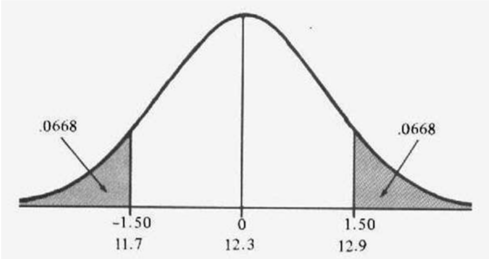
Sampling Error



Sample mean probabilities

- Could μ be 12.3?
- Calculate the z score (11.7 - 12.3)/.4 = - .6/.4 = 1.5,
- Look up this z, area beyond z of 6.68%, doubling gives 13.36% of means falling outside of ± 1.5 (range for means is 12.3 ± .6)
- What do we think about the possibility that μ is 12.3?

Sampling Error (cont.)



(b) $\overline{X}$ deviates 0.6 hours or more from $\mu_{\overline{X}}$; $p = .0668 + .0668 = .1336$

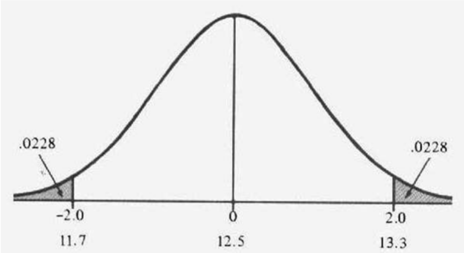
13

Sample mean probabilities

- Could μ be 12.5?
- Calculate the z
  (11.7-12.5)/.4 = -.8/.4 = -2.0
- 2.28% of the area falls beyond
- Doubling gives us 4.56% of means falling at or beyond this point in either direction.
- What do we think about the possibility that μ is 12.5?

14

Sampling Error (cont.)



(c) $\overline{X}$ deviates 0.8 hours or more from $\mu_{\overline{X}}$; $p = .0228 + .0228 = .0456$

15

Population parameters

- General pattern: further sample mean deviates from hypothesized population mean, smaller probability the sample mean is from a population with that mean.
- Hypothesized population means and associated probabilities of a greater deviation than our sample mean of 11.7 hours

Hypothesized Population Mean	Amount of Deviation from μ	Probability of Deviation
12 hours	0.3 hours or more	p = .4532
12.3 hours	0.6 hours or more	p = .1336
12.5 hours	0.8 hours or more	p = .0456

16

ESP example

- ESP example - slips of paper lettered A, B, etc.
- With A & B, only two possible orders, thus 50% chance of being right by chance
- With A, B, C - 6 different orderings:17% chance of getting right. If someone did this would you believe their ESP claim?
- With A, B, C, D – 24 orderings: 4.2% chance of being right
- A, B, C, D, E – 120 orderings, 0.8% chance

17

ESP example (cont.)

- Less likely to conclude that it was simply luck
- What if we only had two letters? How would we test the claim?
- Where do we put the cut off for results being due to chance?

18

Statistical significance (α) (from Ch 9)

- Dictated by where we set alpha level
- Alpha level: beyond this point unwilling to believe that the result is due to random sampling error alone
- Significance levels of .05 or .01 are the most commonly used significance levels
- Levels are completely arbitrary
- Usefulness being seriously questioned
- What characteristic of a data set can have a large impact on whether we attribute results to chance or not?
- An alternative is to focus on effect sizes or magnitude of differences between groups

19

Confidence intervals

- A way of estimating a band of values in which it is likely that the population mean falls
- Sample mean in the middle and can then construct 95% C.I. by adding and subtracting 1.96 x SEM
- 99% C.I. = sample mean $\pm$ 2.58(SEM).
- Where are we getting these numbers?
- Which do we use?

20

Hypothesis testing (One sample case)

- Test some claim about a population parameter
- College women-taller today than they were 10 yrs ago
- Claims avg. ht. of college women is 5’6” (66 in.).
- take random sample of 49 find mean height of 65 inches
- Pop. s.d. for height of women in this age group is 2.5 in.
- So, given this information, can the claim that the population value is 66 inches be correct?
- With what degree of confidence?
- H_0 : null hypothesis. Generally thought of as “no difference”
- In this example: H_0 is μ = 66 in

21

Hypothesis testing example

- Alternative Hypothesis:
  H_1 is $\mu \neq 66$ in
- n = 49, σ = 2.5 in., $\bar{x} = 65in.$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{49}}$ =2.5/7= 0.36
- 95% C.I. = 65 $\pm$ 1.96(.36) = 64.29 to 65.71
- 99% C.I. = 65 $\pm$ 2.58(.36) = 64.07 to 65.93
- So do we accept or reject H_0?

22

Possible sample means

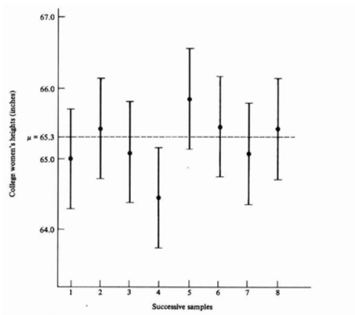


Figure 6.3
Interval estimates of the population mean of college women's heights in successive samples.

23

Hypothesis testing errors

- Sometimes we will be wrong
- How often is related to which C.I. we use
- If 95% expect to be wrong about 5% of the time
- If 99% about 1% of the time.

24

SEM and sample size

- How can we make these C.I.'s narrower?
- Decrease population s.d.
- Increase the n in our sample
- Larger sample gives a better estimate of a population statistic
- Could also increase alpha
- Why not increase alpha?

25

C.I. Width and Sample Size

Table 6.2
Decreasing the 95% confidence interval by increasing the sample size

	Sample $\overline{X} = 60$ **Population $\sigma = 5$**
If $N = 10$,	$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{10}} = \frac{5}{3.1623} = 1.58$ $\overline{X} \pm 1.96\sigma_{\overline{X}} = 60 \pm 3.10 = 56.90$ to 63.10
If $N = 20$,	$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{20}} = \frac{5}{4.4721} = 1.12$ $\overline{X} \pm 1.96\sigma_{\overline{X}} = 60 \pm 2.20 = 57.80$ to 62.20
If $N = 50$,	$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{50}} = \frac{5}{7.0711} = 0.71$ $\overline{X} \pm 1.96\sigma_{\overline{X}} = 60 \pm 1.39 = 58.61$ to 61.39

26

C.I. Width (cont.)

- Two steps further, very precise estimates with larger samples.
- Calculate the following 95% confidence intervals:
  - If n = 100
  - If n = 400

27

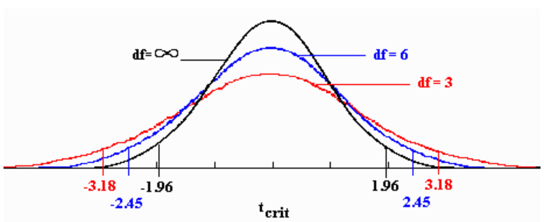
The t Distribution: Defined

- The t distribution is a theoretical probability distribution
- Symmetrical, bell-shaped, and similar to the standard normal curve
- Differs from the standard normal curve, has an additional parameter, called degrees of freedom (df), which changes its shape.
  - can be any real number greater than zero (0.0)
  - Value of df defines a particular member of the family of t distributions
  - t distribution with a smaller df has more area in the tails of the distribution than one with a larger df.

28

Effect of df on the t distribution

- Smaller the df, the flatter the distribution, greater area in the tails of the distribution.



Random samples and research realities

- Every member of population has nonzero probability of being included
- Spatz: all possible samples of size n are equally likely to occur
- Can select using random number table (Table B – 362)
- Generally cannot draw a truly random sample
  - use convenience samples
  - psychology of the UG psychology student
  - random assignment to groups helps us equate our samples

30