

# Basic Statistics

Chapter 7  
Samples, Sampling Distributions, Confidence Intervals (and a first look at hypothesis testing and the t distribution)

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## Outline

- Sampling distributions (sampling distribution of the mean)
- Central limit theorem
- Standard error of the mean
- The z test
- Confidence intervals
- The t distribution
- When to use the normal or t distribution

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## Determining Chance Events

- ESP claims
- Turns over pieces of paper, labeled A and B in correct order
- Convinced?
- A B C?
- How many do you need to see?
- Chance variation vs. significantly different
- Descriptive vs. inferential statistics

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## Descriptive vs. inferential

- Descriptive statistics: simply describing a collection of data
- Inferential statistics: making inferences beyond the data collected
- Describing samples can be interesting
- Usually interested in estimating population parameters
- Sample mean is unbiased estimator of the population mean
- don't know how close we are to true population mean

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## Sampling distributions – basic idea

- Draw all possible random samples of size  $n$  from a population of  $N$
- Calculate the mean of each sample
- Mean of a sampling distribution of means will equal the population mean
- The sampling distribution will be normally distributed with a mean of  $\mu_{\bar{x}}$  and s.d. of  $\frac{\sigma}{\sqrt{n}}$

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## Central limit theorem

- In a population with a mean  $\mu$  and a standard deviation  $\sigma$
- distribution of sample means approaches a normal distribution with a mean  $\mu_{\bar{x}}$  and standard deviation of  $\frac{\sigma}{\sqrt{n}}$
- If the sample size is sufficiently large, the sampling distribution of means will be approximately normal regardless of the shape of the population distribution
- What is sufficiently large?

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## Normal sampling distribution

- A normal distribution, characteristics all apply
- 68% of all sample means will be within  $\pm 1$  s.d. from  $\mu_{\bar{x}}$
- 95% between  $\pm 1.96$  s.d., total of 5% below -1.96 and above 1.96, 2.5% of the area in each tail
- Standard error of the mean = s.d. of the sampling distribution of the mean

- $\sigma_{\bar{x}}$  (or SEM) =  $\frac{\sigma}{\sqrt{n}}$

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## Standard Error of the Mean

- This lets us calculate the probability of various sample means given a certain population mean
- What happens to the SEM as N increases?
- Hours worked per week at part time jobs by 100 college students
- $\bar{x} = 11.7$  and we know that the population s.d.  $\sigma = 4.0$

$$\sigma_{\bar{x}} = \frac{4}{\sqrt{100}} = .4$$

- With a sample mean of 11.7 and SEM = .4 "is it possible" for the population mean,  $\mu$ , to be 12?

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## Sample mean probabilities

- We need a new kind of z score to answer this question
- $x$  = observation
- $\bar{x}$  = sample mean
- $s$  = sample standard deviation
- Replace variables as follows:
- $X$  becomes the sample mean
- $\bar{x}$  becomes the mean of the sampling distribution
- $s$  is replaced with the s.d. of the sampling distribution of the mean (SEM)

$$z = \frac{x - \bar{x}}{s}$$

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## Sample mean probabilities

- $(11.7-12)/0.4 = -.3/.4 = -0.75$
- By sampling error alone, sample means above and below the mean are equally likely
- interested in the probability that a mean could vary this much or more in either direction
- Area under the normal curve beyond a z of .75 in both directions
- Area = 22.66%, doubling = 45.32%
- What would we say about the possibility of a sample mean of 11.7 if true pop. mean is 12?

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## Sample mean probabilities

- Could  $\mu$  be 12.3?
- Calculate the z score  $(11.7 - 12.3)/.4 = -.6/.4 = 1.5$ ,
- Look up this z, area beyond z of 6.68%, doubling gives 13.36% of means falling outside of  $\pm 1.5$  (range for means is  $12.3 \pm .6$ )
- What do we think about the possibility that  $\mu$  is 12.3?

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## Sample mean probabilities

- Could  $\mu$  be 12.5?
- Calculate the z  $(11.7-12.5)/.4 = -.8/.4 = -2.0$
- 2.28% of the area falls beyond
- Doubling gives us 4.56% of means falling at or beyond this point in either direction.
- What do we think about the possibility that  $\mu$  is 12.5?
- Can view graphically.

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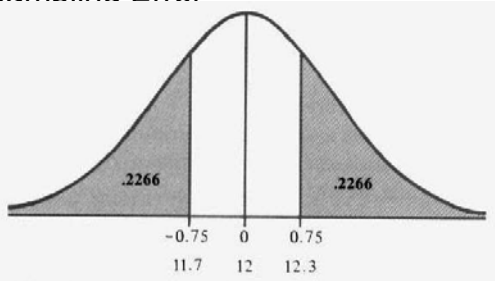
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### Sampling Error



(a)  $\bar{X}$  deviates 0.3 hours or more from  $\mu_{\bar{X}}$ ;  $p = .2266 + .2266 = .4532$

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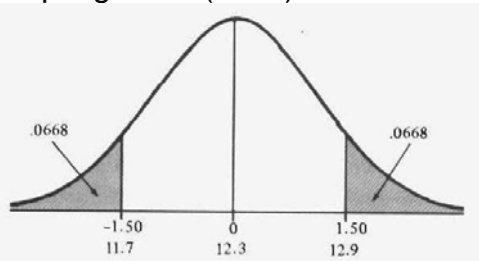
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### Sampling Error (cont.)



(b)  $\bar{X}$  deviates 0.6 hours or more from  $\mu_{\bar{X}}$ ;  $p = .0668 + .0668 = .1336$

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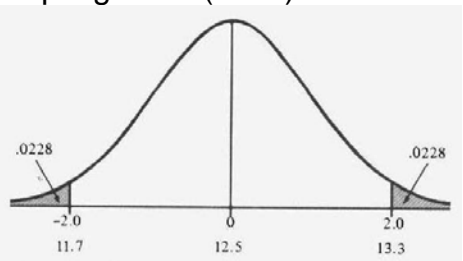
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### Sampling Error (cont.)



(c)  $\bar{X}$  deviates 0.8 hours or more from  $\mu_{\bar{X}}$ ;  $p = .0228 + .0228 = .0456$

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## Population parameters

- General pattern: further sample mean deviates from hypothesized population mean, smaller probability the sample mean is from a population with that mean.
- Hypothesized population means and associated probabilities of a greater deviation than our sample mean of 11.7 hours

Hypothesized Population Mean	Amount of Deviation from $\mu$	Probability of Deviation
12 hours	0.3 hours or more	$p = .4532$
12.3 hours	0.6 hours or more	$p = .1336$
12.5 hours	0.8 hours or more	$p = .0456$

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## ESP example

- ESP example - slips of paper lettered A, B, etc.
- With A & B, only two possible orders, thus 50% chance of being right by chance
- With A, B, C - 6 different orderings: 17% chance of getting right. If someone did this would you believe their ESP claim?
- With A, B, C, D - 24 orderings: 4.2% chance of being right
- A, B, C, D, E - 120 orderings, 0.8% chance

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## ESP example (cont.)

- Less likely to conclude that it was simply luck
- What if we only had two letters? How would we test the claim?
- Where do we put the cut off for results being due to chance?

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## Statistical significance ( $\alpha$ )

- Dictated by where we set alpha level
- Alpha level: beyond this point unwilling to believe that the result is due to random sampling error alone
- Significance levels of .05 or .01 are the most commonly used significance levels
- Levels are completely arbitrary
- Usefulness being seriously questioned
- What characteristic of a data set can have a large impact on whether we attribute results to chance or not?
- An alternative is to focus on effect sizes or magnitude of differences between groups

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## Confidence intervals

- A way of estimating a band of values in which it is likely that the population mean falls
- Sample mean in the middle and can then construct 95% C.I. by adding and subtracting  $1.96 \times \text{SEM}$
- 99% C.I. = sample mean  $\pm 2.58(\text{SEM})$ .
- Where are we getting these numbers?
- Which do we use?

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## Hypothesis testing (One sample case)

- Test some claim about a population parameter
- College women-taller today than they were 10 yrs ago
- Claims avg. ht. of college women is 5'6" (66 in.).
- take random sample of 49 find mean height of 65 inches
- Pop. s.d. for height of women in this age group is 2.5 in.
- So, given this information, can the claim that the population value is 66 inches be correct?
- With what degree of confidence?
- $H_0$  : null hypothesis. Generally thought of as "no difference"
- In this example:  $H_0$  is  $\mu = 66$  in

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## Hypothesis testing example

- Alternative Hypothesis:  
 $H_1$  is  $\mu \neq 66$  in
- $n = 49$ ,  $\sigma = 2.5$  in.,  $\bar{x} = 65$  in.
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{49}} = 2.5/7 = 0.36$
- 95% C.I. =  $65 \pm 1.96(.36) = 64.29$  to  $65.71$
- 99% C.I. =  $65 \pm 2.58(.36) = 64.07$  to  $65.93$
- So do we accept or reject  $H_0$ ?

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## Possible sample means

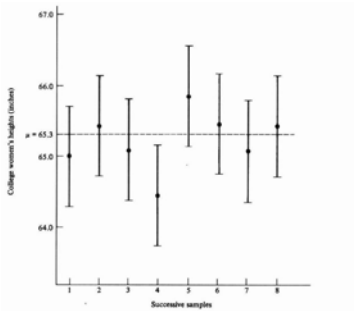


Figure 6.3 Interval estimates of the population mean of college women's heights in successive samples.

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## Hypothesis testing errors

- Sometimes we will be wrong
- How often is related to which C.I. we use
- If 95% expect to be wrong about 5% of the time
- If 99% about 1% of the time.

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## SEM and sample size

- How can we make these C.I.'s narrower?
- Decrease population s.d.
- Increase the n in our sample
- Larger sample gives a better estimate of a population statistic
- Could also increase alpha
- Why not increase alpha?

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## C.I. Width and Sample Size

**Table 6.2**  
Decreasing the 95% confidence interval by increasing the sample size

	Sample $\bar{X} = 60$	Population $\sigma = 5$
If $N = 10$ ,	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{10}} = 1.58$	$\frac{5}{3.1623} = 1.58$
	$\bar{X} \pm 1.96\sigma_{\bar{X}} = 60 \pm 3.10 = 56.90 \text{ to } 63.10$	
If $N = 20$ ,	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{20}} = 1.12$	$\frac{5}{4.4721} = 1.12$
	$\bar{X} \pm 1.96\sigma_{\bar{X}} = 60 \pm 2.20 = 57.80 \text{ to } 62.20$	
If $N = 50$ ,	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = \frac{5}{\sqrt{50}} = 0.71$	$\frac{5}{7.0711} = 0.71$
	$\bar{X} \pm 1.96\sigma_{\bar{X}} = 60 \pm 1.39 = 58.61 \text{ to } 61.39$	

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## C.I. Width (cont.)

- Two steps further, very precise estimates with larger samples.
- Calculate the following 95% confidence intervals:
  - If  $n = 100$
  - If  $n = 400$

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## The t Distribution: Defined

- The t distribution is a theoretical probability distribution
- Symmetrical, bell-shaped, and similar to the standard normal curve
- Differs from the standard normal curve, has an additional parameter, called degrees of freedom (df), which changes its shape.
  - can be any real number greater than zero (0.0)
  - Value of df defines a particular member of the family of t distributions
  - t distribution with a smaller df has more area in the tails of the distribution than one with a larger df.

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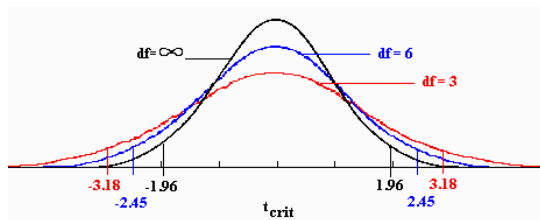
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## Effect of df on the t distribution

- Smaller the df, the flatter the distribution, greater area in the tails of the distribution.



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## Random samples and research realities

- Every member of population has nonzero probability of being included
- Spatz: all possible samples of size n are equally likely to occur
- Can select using random number table
- Generally cannot draw a truly random sample
  - use convenience samples
  - psychology of the UG psychology student
  - random assignment to groups helps us equate our samples

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