Applied Statistics for the Behavioral Sciences

Chapter 4
Variability

Chapter 4 Outline

- Measures of Variability
  - range (including interquartile)
  - standard deviation(s)
  - variance
- Graphical presentation

Variability/Dispersion

- Another characteristic of a distribution
- Frequency tables and graphs show us the form
- Measures of central tendency: location
- Measures of dispersion: variation or spread
- Three estimates:
  - Range
  - Standard Deviation
  - Variance

Range (incl. Interquartile)

- Range = distance between high & low scores
- Administer the Beck Depression Inventory to a group of students
- Scores: 15,20,21,20,36,16, 25,15
- The range would be 36-15
- 21 points separate the highest and lowest scores
- range = X_H - X_L
- Interquartile range
  - distance between the 25th and 75th percentiles
  - benefit relative to range?

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Standard Deviation

- The Standard Deviation is a value that shows the relation that individual scores have to the mean of the sample.
- Can be thought of as roughly the average deviation from the mean
  - more accurate/detailed
  - outlier(s) can greatly extend the range
  - Standard deviation uses info from all of the data
  - not dictated solely by the extreme values

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Standard Dev. (con't)

- Three different SDs - different purposes
  - σ measure population variability
  - $\hat{s}$ estimate population variability
  - S measure sample variability
- σ & S are considered descriptive indices
  - calculated the same way
  - use different terms in equations

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Standard Deviation (con't)

$$\sigma = \sqrt{\frac{\sum (X-\mu)^2}{N}} \qquad S = \sqrt{\frac{\sum (X-\overline{X})^2}{N}}$$

- **σ = population standard deviation**
- **S = sample standard deviation**
- **X = a data point (observation)**
- **N = number of deviations (observations)**

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Standard Dev. (con't)

- Generally interested in standard deviation as estimator
- Dividing the sum of squared deviations by N has been shown to result in a significantly biased estimate of σ
- This estimate is generally too small, underestimating the population parameter
- Using N-1 results in a better estimate

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Standard Dev. (con't)

- The standard deviation is the square root of: the sum of the squared deviations from the mean of all the scores divided by the number of scores - 1

$$\hat{s} = \sqrt{\frac{\Sigma(X-\overline{X})^2}{n-1}}$$

Definitional Formula

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Example Standard Deviation

- Lets take the set of scores: 15,20,21,20,36,16, 25,15
- The mean of this sample is 21, first line up the scores
- 15,15,16,20,20,21,25,36.
- 15-21=-6, 15-21=-6, 16-21=-5, 20-21=-1, 20-21=-1, 21-21=0, 25-21=4, 36-21=15
- Square these values: -6, -6, -5, -1, -1, 0, 4, 15
- Add them all up: 36+36+25+1+1+0+16+225 = 340

Working Ex. (Stan. dev. con't)

- Divide 340 by 7 = 48.57
- square root of 48.57 = 6.97
- 6.97 = Standard Deviation

Computational Formula

$$\sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N-1}}$$

SD Computational Formula

- Take values, square each
- Sum both columns
- Square the summed X's
- Plug into formula

$$\hat{s} = \sqrt{\frac{3868 - \frac{28224}{8}}{8-1}}$$

	X	X^2
	15	225
	15	225
	16	256
	20	400
	20	400
	21	441
	25	625
	36	1296
Σ =	168	3868
	28224	

SD Computational Formula

$$= \sqrt{\frac{3868 - 3528}{7}}$$

$$= \sqrt{\frac{340}{7}}$$

$$= \sqrt{48.57}$$

$$= \mathbf{6.9693}$$

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Graphical presentation of central tendency & variability

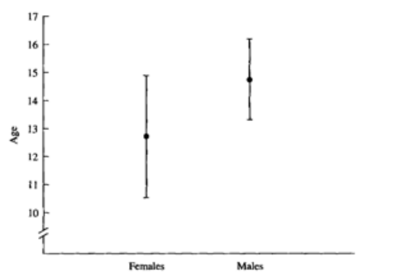

FIGURE 3.1 Puberty in females and males—means and standard deviations

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Variance

- The square of the standard deviation
- If we subtract each value from the mean and take their average, always zero, just as many values are above as below the mean
- Squaring differences eliminates the problem of summing to zero
- Average squared deviation is the variance

$$\frac{\Sigma(X - \overline{X})^2}{n - 1}$$

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Variance Computational Formula

X	X²
0	0
1	1
1	1
1	1
2	4
2	4
2	4
2	4
2	4
4	16
ΣX=17,(ΣX)²=289	ΣX² = 39

$$\hat{s}^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N-1}$$

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Variance Computational Formula

X	X²
0	0
1	1
1	1
1	1
2	4
2	4
2	4
2	4
2	4
4	16
ΣX=17,(ΣX)²=289	ΣX² = 39

$$\hat{s}^2 = \frac{39 - \frac{289}{10}}{n-1}$$

$$\hat{s}^2 = \frac{39-28.9}{9} = \frac{10.1}{9} = 1.12$$

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