


Advanced Experimental Design

Topic 8
Chapter 14: Repeated Measures Analysis of Variance


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Overview

- Basic idea, different forms of repeated measures
- Partialling out between subjects effects
- Simple repeated measures ANOVA
 - structural model
- New assumptions
 - Sphericity & equality of covariance matrices
- Mixed model (adding a between subjects factor)
- SPSS example(s)

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Repeated measures ANOVA

- Similar to switch from independent samples t to paired samples t-test
- Basic design consists of the same subject being measured on all levels of DV
 - different conditions
 - measures taken over time
- Advantage: reduces the overall variability within subjects; allows removal of subject differences from the error term

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Different forms of repeated measures

- Time – same measurement at multiple points in time
- Different conditions
- Different versions of the same measuring device
- Matched triads or larger groups

- Potential biases
 - Practice effects
 - Fatigue
 - Confounding effects

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Partitioning Variability - Oneway

- In oneway ANOVA, between groups and within groups

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graph TD
    A[Total SS] --> B[Between Groups SS]
    A --> C[Within Groups SS]
  
```

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Partitioning Variability – Repeated Measures

- In Repeated Measures ANOVA
- Between subjects SS
- Within subjects variability
 - between treatments variance
 - error variance - individual variability in impact of treatment
- Repeated measure over more than two levels
- Hypothesis test focuses on change over time (or across treatments)

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Partitioning Variability – Repeated Measures

- SS divided into between subjects and within subjects, which is then divided further

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graph TD
    A[Total SS (kn-1)] --> B[Between subjects (n-1)]
    A --> C[Within subjects (n(k-1))]
    C --> D[Between treatments (k-1)]
    C --> E[Error ((n-1)(k-1))]
  
```

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Partialling out subject effects

- each subjects contribution to the overall variability is placed on a common footing
- Howell Table 14.1, Pg. 441
- Each subject's scores expressed as deviations from their overall mean score

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Partialling out subject effects

Subject	Treatment			Mean	Sub	1	2	3	M
	1	2	3						
1	2	4	7	4.33	1	-2.33	-0.33	2.67	0.00
2	10	12	13	11.67	2	-1.67	0.33	1.33	0.00
3	22	29	30	27.00	3	-5.00	2.00	3.00	0.00
4	30	31	34	31.67	4	-1.67	-0.67	2.33	0.00
Mean	16	19	21	18.67	M	-2.67	0.33	2.33	0.00

Original	12.44	13.14	13.04
Deviations	1.59	1.19	0.72

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Structural model for repeated measures ANOVA

- Represents each individual score for each treatment in terms of the model parameters
- Used to derive expected mean squares for the different portions of the total variance
- F ratios that we will use to evaluate our null hypothesis

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Structural model

$$X_{ij} = \mu + \pi_i + \tau_j + \pi\tau_{ij} + e_{ij}$$

- μ = grand mean
- π_i = a constant associated with the i^{th} person
- τ_j = a constant associated with the j^{th} treatment
- $\pi\tau_{ij}$ = the subject by treatment interaction
- e_{ij} = the experimental error for the i^{th} person under the j^{th} treatment

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Assumptions

- Compound symmetry of the covariance matrix
- covariance matrix = unstandardized correlation matrix
 - Variances on the diagonal, off diagonal elements are covariances between variables

$$\Sigma = \textit{population}$$

$$\hat{\Sigma} = \textit{estimate}$$

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Covariance

- Crossproducts of deviations from the mean
- Dividing by product of sd's turns into a correlation coefficient

$$COV_{xy} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{N - 1}$$

$$\text{or } \frac{\sum XY - \frac{\sum X \sum Y}{N}}{N - 1}$$

$$r = \frac{COV_{xy}}{S_x S_y}$$

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Compound symmetry/Sphericity

- Compound symmetry: variances all about the same; covariances all about the same
 - variances and covariances do not need to be similar
- Sphericity: less restrictive condition, which can be present in the absence of compound symmetry
- repeated-measures ANOVA equivalent of the homogeneity of variances assumption
- SPSS provides Mauchly's test of Sphericity
- Run example from Table 14.3 in text in SPSS

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Mixed model

- add a grouping factor
- a little more complicated
- partitioning the variance in our DV
- in both major partitions, the S's within groups term represents the "error" or residual term

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Partitioning Variability – mixed model, 1 between and 1 within

- Three hypothesis tests
 - group, interval, interval x group

i = intervals
 g = groups
 n = sample size

Adapted from Howell (2007) pg 451
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Another assumption

- equality of covariance matrices across groups
- interrelationships among intervals of the DV do not differ across groups
- SPSS provides us with Box's test of the equality of covariance matrices
- Mixed model example in SPSS, Table 14.4 data from text

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