Advanced Experimental Design

Topic 7

Chapter 13: Factorial Analysis of Variance (ANOVA)

Factorial ANOVA

- More new terms
- Main effects & interactions
- Cell means & marginal means
- Null hypotheses
- Computational example
- Summary table
- SPSS example
- Magnitude of Effect

New terms

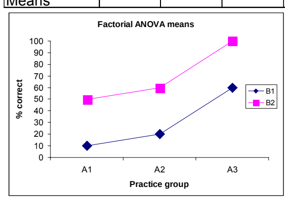
- factor - an independent variable
- level - a different treatment or group
- factorial design-design where two or more independent variables each has two or more levels
- cell - a unique way of treating some smaller group of participants

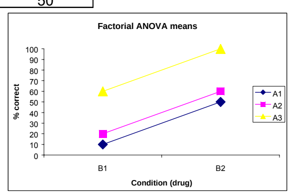
New terms (cont.)

- main effect - the effect that one factor has ignoring the other
- interaction effect - if significant basically says that the effect of one factor varies by level of another factor
- Partitioning between subjects variability into main effects and an interaction term

Cell means/Marginal means

	A1	A2	A3	Factor B Means
B1	10	20	60	30
B2	50	60	100	70
Factor A Means	30	40	80	Gnd. Mn. 50



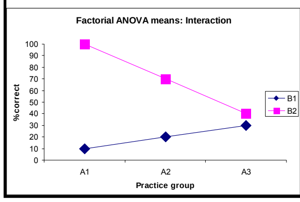


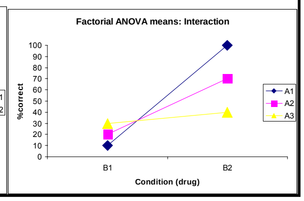
3 Null hypotheses

- in two-way ANOVA we have three different null hypotheses
  - AB interaction
  - main effect of A
  - main effect of B
- interaction represents the variability that is "left over" after the main effects have been removed
- Substitute variable names for A, B, etc.

Significant interaction

	A1	A2	A3	Factor B Means
B1	10	20	30	20
B2	100	70	40	70
Factor A Means	55	45	35	Gnd. Mn. 45

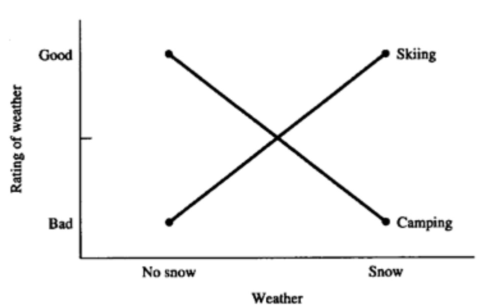




Interactions in ANOVA

- can have a statistically significant interaction in the absence of a significant main effect
- plot a line graph of the means
- difficult to get a clear idea of pattern from table of means

Interactions in ANOVA



Higher order designs

- principle is essentially the same for any higher order factorial ANOVA
- for a three-way ANOVA we will have seven null hypotheses to test
  - AxBxC, AxB, AxC, and BxC interactions and the A, B, & C main effects
- In all cases, same multiple comparison tests will be used to examine main effects further

Degrees of freedom

- Same basic idea as Oneway ANOVA, simply adds the df for the interaction term
- main effects will be the number of levels – 1
- interaction df = product of the df's for the main effects involved in the interaction
- generalizes to more complex designs
- total df will still be n – 1
- error df = total df – summed df for all main effects and interactions in the model (or n-1/cell multiplied by n of cells)

Calculations/practice problem

- Illustration of computations for factorial ANOVA
- Work through by hand, follow along on handout
- Example looks at need for closure as a function of illumination and anxiety
- What kind of design is this? df?
- n_{ijk} *i=row, j=column, k=case*

Calculations for Factorial ANOVA

$$SS_{total} = \sum \left(X - \bar{X}_{..} \right)^2$$

$$SS_{illum} = nA \sum \left(\bar{X}_{i.} - \bar{X}_{..} \right)^2$$

$$SS_{anx} = nI \sum \left(\bar{X}_{.j} - \bar{X}_{..} \right)^2$$

$$SS_{cells} = n \sum \left(\bar{X}_{ij} - \bar{X}_{..} \right)^2$$

$$SS_{ia} = SS_{cells} - SS_{illum} - SS_{anx}$$

$$SS_{error} = SS_{total} - SS_{cells}$$

Cell and marginal means

Anxiety

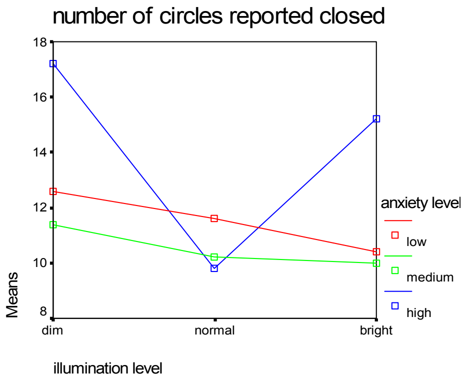
Cell mns	low (1)	med (2)	hi (3)	row mns
dim (1)	12.60	11.40	17.20	13.73
normal (2)	11.60	10.20	9.80	10.53
bright (3)	10.40	10.00	15.20	11.87
col mns	11.53	10.53	14.07	**12.04**

Gr mean

ANOVA Summary Table

Source	df	SS	MS	F
I (Illumination)	2	77.51	38.76	7.27*
A (Anxiety)	2	99.51	49.76	9.33*
I x A interaction	4	86.89	21.72	4.07*
Error	36	192	5.33	
Total	44	455.91		*p<.01

Examining Interactions: means plot



Simple effects

- Does one variable have an effect at some level of another variable?
- Removes variable not being considered from the $SS_{between}$ calculation
- SS_{error} remains same, best estimate with most df

Simple effects of interest

- anxiety levels at normal illum
- illum levels at low & medium anxiety levels

$$SS_{anx-at-normal} = n\sum(\bar{X}_{2j} - \bar{X}_{2\cdot})^2$$

$$SS_{illum-at-medium} = n\sum(\bar{X}_{i2} - \bar{X}_{\cdot 2})^2$$

$$SS_{illum-at-low} = n\sum(\bar{X}_{i1} - \bar{X}_{\cdot 1})^2$$

Magnitude of Experimental Effect

- Eta squared, as presented by Howell, again overestimates effect in population
- Omega squared is a better estimator of population effects
- How various terms are derived will depend on design
- See Pg. 413-416 (Howell)

Calculating ω^2

$$X_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + e_{ijk}$$

$$\hat{\omega}^2_{effect} = \frac{\hat{\sigma}^2_{effect}}{\hat{\sigma}^2_{total}}$$

$A_f B_f$

$$\hat{\sigma}^2_{\alpha} = (a-1)(MS_A - MS_e)/nab$$

$$\hat{\sigma}^2_{\beta} = (b-1)(MS_B - MS_e)/nab$$

$$\hat{\sigma}^2_{\alpha\beta} = (a-1)(b-1)(MS_{AB} - MS_e)/nab$$

$$\hat{\sigma}^2_{e} = MS_e$$