Advanced Experimental Design

Topic 5

T-test Computations, Effect Size Index, & Power Calculations

1

Agenda

- t-test computations
  - 1 sample
  - Paired sample
  - Independent sample
- Effect size index
- Power calculations
  - Using Delta & Power table
  - Using G-Power

2

One sample t-test

- uses the characteristics of the t-distribution to account for the number of subjects in the calculations of $s_{\overline{x}}$
- would be used when the population standard deviation is unknown

$$t = \frac{\overline{X} - \mu_0}{s_{\overline{x}}}$$

3

Paired samples t-test

- measuring individuals at different points in time
- assess whether meaningful change has occurred over time can be answered with a paired samples t-test
- can also look at other types of paired differences

4

Paired samples t-test (cont.)

- paired samples t statistic is a ratio of the pre-post (or time1-time2) mean differences divided between the standard error of the mean differences
- This t-test has n_{pairs} - 1 degrees of freedom
- Example: Cholesterol readings before and after 4 weeks of treatment with diet & exercise

5

Computation of paired t

Cholest1	Cholest2	Difference (D)	D^2
260	240	-20	400
255	260	5	25
270	255	-15	225
300	235	-65	4225
315	270	-45	2025
280	230	-50	2500
265	285	20	400
295	225	-70	4900
250	200	-50	2500
305	265	-40	1600

$\Sigma D = -330$ $\Sigma D^2 = 18800$ $(\Sigma D)^2 = 108900$

$M_D = -33$ $s^2_D = 878.89$ $s_D = 29.65$

$\sigma_{seD} = 9.37$

t = MD/S.E. Diff, t = -3.52

$t_{crit}(9)$, alpha =.05, 2-tailed = 2.26

$$t = \frac{\bar{X} - \bar{Y}}{s_{\bar{D}}} = \frac{\bar{X} - \bar{Y}}{s_D / \sqrt{N}}$$

$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{N}}{N-1}}$$

$$s_{\bar{D}} = \sqrt{s_{\bar{X}}{}^2 + s_{\bar{Y}}{}^2 - 2r_{XY}(s_{\bar{X}})(s_{\bar{Y}})}$$

6

Computation of independent samples t

same data, but in this case we treat the repeated measures as though they were two different conditions at post-test

Group	Chol
No Exercise	260
No Exercise	255
No Exercise	270
No Exercise	300
No Exercise	315
No Exercise	280
No Exercise	265
No Exercise	295
No Exercise	250
No Exercise	305
Exercise	240
Exercise	260
Exercise	255
Exercise	235
Exercise	270
Exercise	230
Exercise	285
Exercise	225
Exercise	200
Exercise	265

7

$$s_{\bar{x}_1-\bar{x}_2}=\sigma_{se}$$

	No Exercise	X^2	Exercise	X^2
	260	67600	240	57600
	255	65025	260	67600
	270	72900	255	65025
	300	90000	235	55225
	315	99225	270	72900
	280	78400	230	52900
	265	70225	285	81225
	295	87025	225	50625
	250	62500	200	40000
	305	93025	265	70225
ΣX	2795	785925	2465	613325
$(\Sigma X)^2$	7812025		6076225	
ΣX^2	785925		613325	
s^2	524.72		633.61	
M	279.5		246.5	
M1-M2	33			
σ_{se}	10.76			
t=M1-M2/σ_{se}				
t=	3.07			
t_{crit}(18), .05, 2tail=	2.101			

Independ. samples t (cont.)

- Where $s_{\bar{x}_1-\bar{x}_2}$ $\sqrt{\dfrac{s_1^{\,2}+s_2^{\,2}}{n}}$
- since the sample sizes for the two groups are the same
- If they differed, the formula would be:

$$\sqrt{\frac{(n_1-1)s_1^{\,2}+(n_2-1)s_2^{\,2}}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

9

Combining repeated measures and independent samples t tests

- question that involves change over time, in response to some sort of treatment, education, or intervention
  - combine these two designs into a two group pretest-posttest design.
- Treatment and a control group
  - Calculate difference scores for each subject
  - Carry out an independent samples t-test on these difference scores.

10

Effect size

- Cohen's d statistic, one-sample case simplest to calculate
- Interpretation is identical regardless
- Tells us how much of an effect the independent variable had

$$d = \frac{|\bar{X} - \mu_0|}{\sigma}$$

11

Effect size index

- Independent samples $$d = \frac{|\bar{X}_1 - \bar{X}_2|}{\hat{s}}$$
- if N1 = N2, $$\hat{s} = \sqrt{N_1}(s_{\bar{x}_1 - \bar{x}_2})$$
- If N1 ne N2, $$\hat{s} = \sqrt{\frac{\hat{s}_1^{\,2}(df_1) + \hat{s}_2^{\,2}(df_2)}{df_1 + df_2}}$$

Effect size index

- For correlated samples, same general formula:

$$d = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{\hat{s}}$$

- Difference is in $\hat{s} = \sqrt{N}(s_{\bar{D}})$

13

Lazarus study

- 2 groups of participants watched a film that showed accidents occurring in a workshop.
- The accidents were gruesome events such as fingers being cut off and a plank being thrown through a man's midsection by a circular saw.
- One group was instructed to remain detached from the events. The other group was instructed to become involved.
- Heart rate was monitored and increases noted.

14

Unequal n example t-test

- Lazarus data
- Do by hand
- t-test & effect size
- Formulate conclusion

Detached	Involved
23	31
21	27
19	24
15	23
14	21
12	14
10	

15

Estimating power / sample size "manually"

- Howell uses delta as a way of using one power table for all of the various tests
- δ (delta): function of effect size and some function of n (function of n defined specifically by test)

$$\delta = d\left[f\left(n\right)\right]$$

- Solve for delta and look up power, Cohen's book has many, many pages of power tables

16

Power examples

$$\delta = d\sqrt{n}$$

- One sample t
- Class has higher SAT scores (545) than nat. avg. = 500, sigma = 100, 40 students
  - What is our power?
- Finding n to attain given power
  - For SAT example for power = .8
  - What if difference was only 20 pts?

$$n = \left(\frac{\delta}{d}\right)^2$$

17

Power calculations

- Independent t, with equal n/group

$$\delta = d\sqrt{\frac{n}{2}}$$

- n = n/group
- Solving for n for a given power

$$n = 2\left(\frac{\delta}{d}\right)^2$$

18

Noncentrality Parameter

- With known μ, the sampling distribution of t will be centered around 0 regardless of the status of H_0.
- If H_0 is false, and we're actually using wrong population mean, distribution will be centered at delta (δ)
- incorporates d and n
- split noncentrality parameter into constituent parts, the effect size and the sample size

$$\delta = \frac{\mu_1 - \mu_0}{\sigma / \sqrt{n}}$$

19

Power calculations

- independent t, with unequal n
- harmonic mean of the group sizes is used
- where k is number of elements and Xs are the individual numbers

$$\bar{X}_h = \frac{k}{\sum \frac{1}{X_i}}$$

$$\bar{n}_h = \frac{2}{\frac{1}{n_1} + \frac{1}{n_2}} = \frac{2n_1 n_2}{n_1 + n_2}$$

$$\delta = d\sqrt{\frac{\bar{n}_h}{2}}$$

20

G-Power

- Free program
- Makes it even easier to carry out power analyses
- Can check out multiple scenarios for your research very quickly
- http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/

21