PSY 464
Advanced Experimental Design

Describing and Exploring Data
The Normal Distribution

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Overview/Outline

- Questions-problems?
- Exploring/Describing data
  - Organizing/summarizing data
  - Graphical presentations
    - Histogram, Stem and leaf plot, & Boxplot
  - Describing Distributions
  - Central tendency
  - Variability
- Normal Distribution
  - Description, Z-scores, Areas & Probabilities

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Moving beyond raw data

- Unorganized/interpretable
- Imposing organization
- Ordering
- N = total number of observations (scores)
- *f* = frequency of each score

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Y-DACL Scores – File order

0	13	0	4	7	2	0	15	3	6	1	2	8	3	15
6	4	5	16	5	10	4	1	1	6	8	4	4	6	12
4	8	21	6	8	6	6	6	4	3	8	3	19	12	4
6	0	1	8	4	7	6	5	1	7	12	2	4	11	6
5	5	1	5	7	5	8	7	4	8	6	5	8	10	15
9	8	7	4	0	1	1	3	0	13	0	8	10	7	13
7	2	4	14	4	6	15	6							

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Y-DACL Scores – Ordered

0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
2	2	2	2	3	3	3	3	3	4	4	4	4	4	4
4	4	4	4	4	4	4	4	5	5	5	5	5	5	5
5	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	8	8	8	8	8	8	8
8	8	8	8	9	10	10	10	11	12	12	12	13	13	13
14	15	15	15	15	16	19	21							

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Simple Frequency Table

Y-DACL	f	Y-DACL	f
0	7	12	3
1	8	13	3
2	4	14	1
3	5	15	4
4	14	16	1
5	8	17	0
6	14	18	0
7	8	19	1
8	11	20	0
9	1	21	1
10	3	22	0
11	1	missing	2
		n=	100

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Grouped Frequency Table

Y-DACL interval	midpoint	f	cumulative f
0-2	1	19	19
3-5	4	27	46
6-8	7	33	79
9-11	10	5	84
12-14	13	7	91
15-17	16	5	96
18-20	19	1	97
21-23	22	1	98

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Frequency table from SPSS

ydacl

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	.00	7	7.0	7.1	7.1
	1.00	8	8.0	8.2	15.3
	2.00	4	4.0	4.1	19.4
	3.00	5	5.0	5.1	24.5
	4.00	14	14.0	14.3	38.8
	5.00	8	8.0	8.2	46.9
	6.00	14	14.0	14.3	61.2
	7.00	8	8.0	8.2	69.4
	8.00	11	11.0	11.2	80.6
	9.00	1	1.0	1.0	81.6
	10.00	3	3.0	3.1	84.7
	11.00	1	1.0	1.0	85.7
	12.00	3	3.0	3.1	88.8
	13.00	3	3.0	3.1	91.8
	14.00	1	1.0	1.0	92.9
	15.00	4	4.0	4.1	96.9
	16.00	1	1.0	1.0	98.0
	19.00	1	1.0	1.0	99.0
	21.00	1	1.0	1.0	100.0
	Total	98	98.0	100.0	
Missing	System	2	2.0		
Total		100	100.0		

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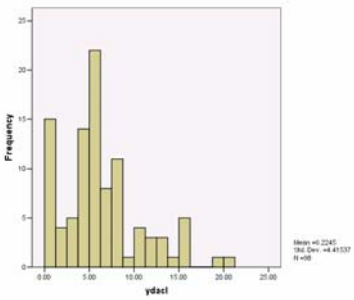
Histogram

- appropriate for quantitative data



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Stem and Leaf Plot

- Little or no loss of information

```
Freq    Stem &  Leaf
 7.00     0 .  0000000
 8.00     1 .  00000000
 4.00     2 .  0000
 5.00     3 .  00000
14.00     4 .  00000000000000
 8.00     5 .  00000000
14.00     6 .  00000000000000
 8.00     7 .  00000000
11.00     8 .  00000000000
 1.00     9 .  0
 3.00    10 .  000
 1.00    11 .  0
 3.00    12 .  000
 3.00    13 .  000
 8.00 Extremes    (>=14.0)
Stem width:      1.00
Each leaf:       1 case(s)
```

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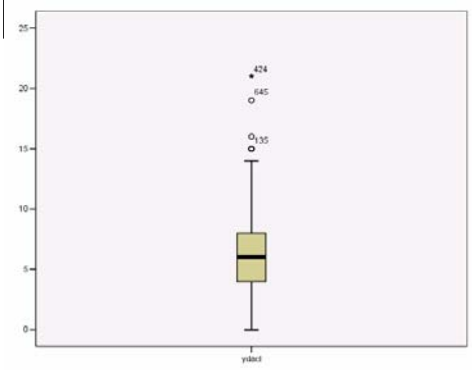
Boxplot



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Describing distributions

- Normal distribution: Bell shaped curve
  - most observations concentrated in middle
  - very well-known properties.
- Skewed distributions
  - distribution is not symmetrical
  - tail trails off in one direction or the other
  - greatest frequency of observations not in middle
  - skewness is in which direction?
- Bimodal
- Kurtosis

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Measures of central tendency

- mean
- median
- mode

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Mean

- a statistic calculated from a sample
- corresponding population parameter is μ
- population parameter, we know the exact value with certainty
- statistic uncertainty is involved
- $\overline{X}$ is the best estimator of μ

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Summation notation

- Σ uppercase Greek sigma
- tells us to add up an entire group of numbers
- ΣX means add up all the Xs
- Page 32-33 Summation Notation
- $(\Sigma X)^2$ vs. ΣX^2

$$\sum X$$

$$\sum X^2$$

$$\left(\sum X\right)^2$$

$$\sum XY$$

$$\sum (X - Y)$$

$$\left(\sum X\right)\left(\sum Y\right)$$

$$\sum (X - Y)^2$$

X	Y
1	2
3	0
0	3
2	4
4	2

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Formula for the mean

- where:
- $\overline{X}$ = the mean
- ΣX = add up all the X values
- N = number of scores

$$\overline{X} = \frac{\Sigma X}{N}$$

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Median

- point at the exact middle of the set of scores.
- list all scores in numerical order, and then locate the point in the center of the sample.
- Median = location (N+1)/2
- simplest interpretation it is the score or value where half are higher and half lower
- 50th percentile of a set of scores

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Mode

- simply the most frequently occurring value in a set of scores
- when giving a modal value should also give an idea of how often it occurred
- can be more than one mode
- if multiple modes, simply list all

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Which should I use?

- scale of measurement dictates measure of central tendency
- mean with interval/ratio level data, not ordinal/nominal.
- median with ordinal or higher, not nominal.
- mode with any level

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Skewness

- If the distribution is normal (i.e., bell-shaped), the mean, median and mode are all about equal
- positive skew: Mean>Median>Mode
- negative skew: Mean<Median<Mode
- means from highly skewed distributions can be misleading

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Variability

- Dispersion around the middle of the distribution, generally the mean
  - Range
  - Interquartile range
  - Variance
  - Standard deviation

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Range and IQR

- Range
  - distance between the highest and lowest scores
  - completely dictated by extreme values
- Interquartile range
  - distance between the 25th & 75th percentile
  - completely ignores extreme values

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Variance

- If we subtract each value from the mean and take their average, it always comes out to zero, values above and below the mean cancel each other out
- Squaring each of these differences eliminates the problem of summing to zero
- The average squared deviation is the variance

$$\frac{\Sigma(X-\overline{X})^2}{n-1}$$

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Standard Deviation

- Variance provides us with average squared deviation from the mean
- Taking square root, essentially gets us back to our original measurement scale
  - Definitional form
  - Computational form.

$$\sqrt{\frac{\Sigma(X-\overline{X})^2}{n-1}} \qquad \sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}}$$

Descriptive statistics, from descriptives & frequencies

Descriptive Statistics

		Statistic	Std. Error
ydacl	N	98	
	Range	21.00	
	Minimum	.00	
	Maximum	21.00	
	Sum	610.00	
	Mean	6.2245	.44602
	Std. Deviation	4.41537	
	Variance	19.495	
	Skewness	.956	.244
	Kurtosis	.957	.483
Valid N (listwise)	N	98	

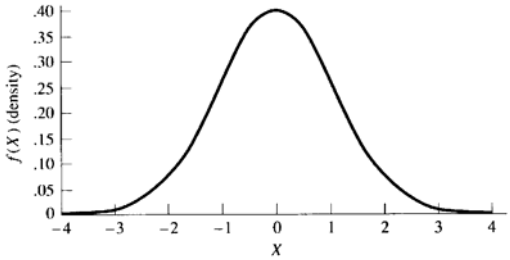
Statistics

ydacl

N	Valid	98
	Missing	2
Mean		6.2245
Std. Error of Mean		.44602
Median		6.0000
Mode		4.00[a]
Std. Deviation		4.41537
Variance		19.495
Skewness		.956
Std. Error of Skewness		.244
Kurtosis		.957
Std. Error of Kurtosis		.483
Range		21.00
Minimum		.00
Maximum		21.00
Sum		610.00
Percentiles	5	.0000
	25	3.7500
	50	6.0000
	75	8.0000
	95	15.0000

a. Multiple modes exist. The smallest value is shown

Normal distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}(e)^{-(x-\mu)^2/2\sigma^2}$$

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Standard Normal Curve

- μ = 0.0 and σ = 1.0
- 1.0 for σ, simply a unit value
- X-axis on a standard normal curve is often relabeled and called Z scores
- Howell: page 694-697, table of areas under normal curve-*appendix Z*

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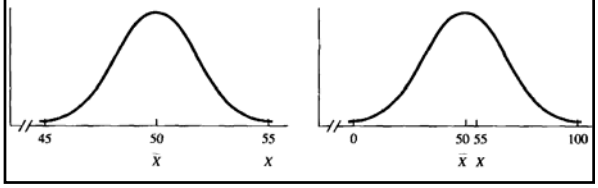
Standard Scores

- If we want to directly compare standing on measures with different scales



Standard Scores

- Even with same mean, distributions may be very different
- standard scores: convert to common scale
- z-scores: standard scores expressed in standard deviation units

$$z = \frac{X - \overline{X}}{S}$$

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A problem

- ***Lets compare two cases in terms of relative standing on level of depression***
- ***A tricky twist – missing data***

idscana	rads	ydacl
151	2.41	-
1248	-	8.00

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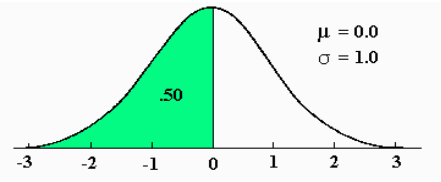
Normal Curve (cont.)

- total area below 0.0 is .50
- symmetrical about the mean, thus area above 0.0 is .50
- generalizes to all normal curves: total area below the value of m is .50 on any normal curve



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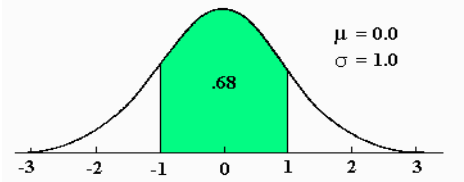
Normal Curve (cont.)

- area between Z-scores of -1.00 and +1.00. It is .68 or 68%
- total area between plus and minus one sigma unit (1 s.d.) on any normal curve is also .68.



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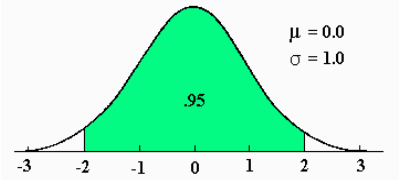
Normal Curve (cont.)

- area between Z-scores of -1.96 and +1.96 (about 2) is .95 or 95%
- area (.95) generalizes to plus and minus two sigma units on any normal curve



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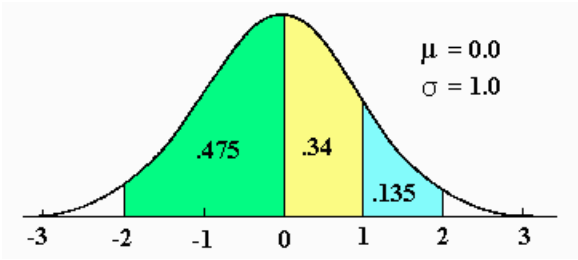
Finding Areas (cont.)



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Areas under Normal Curve

- area below a Z-score of 1.0?
- computed by adding .34 and .50 to get .84
- area above a Z-score of 1.0?
- subtract the area just obtained from the total area under the distribution (1.00)
- 1.00 - .84 or .16 or 16%

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Areas under Normal Curve

- area between -2.0 and -1.0?
- first, the area between 0.0 and -2.0 is 1/2 of .95 or .475
- the .475 includes too much area, the area between 0.0 and -1.0 (.34) must be subtracted from this
- so, .475 - .34 or .135

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Using normal curve

- Mean = 10, SD = 2
  - what proportion of scores is between 7.5 & 12.5?
  - what proportion of scores is between 7.5 & 10.5?
  - What score separates the lower 40% from the upper 60%?
  - If there were 250 members of population, how many would be expected to score 11 or more?
  - What proportion would be expected to score 9 or more?
  - What score separates the top 10% of scores from the rest?

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