Advanced Experimental Design
Psych 464
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Topic 1: Correlation / Linear Regression

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Outline/Overview

- Correlations (r, pr, sr)
- Linear regression
- Multiple regression
  - interpreting b coefficients
  - ANOVA model test
  - R^2
  - Diagnostics
    - Distance(Discrepancy), Leverage, Influence, & Multicollinearity

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The correlation coefficient (Pearson r)

- Continuous IV & DV (Interval/Ratio level data)
- Dichotomous variables
- Strength of linear relationship between variables
- r^2 coefficient of determination

$$r = \frac{\Sigma Z_x Z_y}{N-1}$$

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Partial and semipartial correlations

- Partial Correlation
  - r between two variables when one or more other variables is partialled out of both X and Y
  - example: experimenter interested in investigating the relationship between income and success in college
    - measured both variables, ran correlation
    - it was statistically significant
- he began repeating these results to all of his students
- do well in college = large salaries
- what might the bright student in the back of one of his classes have raised as an issue?

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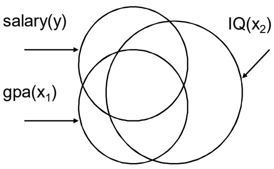
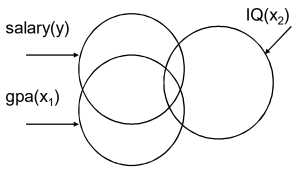
Partial correlation (cont.)

- IQ as a third variable effecting both college success and salary earned
- The partial correlation between college success and salary with IQ partialled out of both variables





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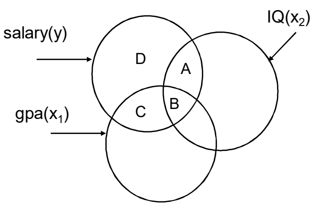
Partial correlation (cont.)

- correlation between the residuals is the partial correlation between income and success, partialling out IQ
- generally represented by $r_{y1.23...p}$
  - subscripts to the left of the dot represent variables being correlated
  - those to the right of the dot are those being partialled out
- $r_{y1.2}$=correlation between salary and GPA with IQ partialled out
- $pr^2=c/(c + d)$



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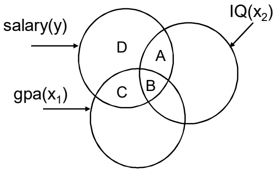
Semipartial correlation

- also called the part correlation, far more routinely used
- $r_{y(1.2)}$ represents the correlation between the criterion (y) and a partialled predictor variable
- the partial correlation has variable x_2 partialled out of both y and x_1
- for the semipartial, x_2 is partialled only out of x_1
- the correlation between y and the residuals from x_1 predicted by x_2
- correlation between y and the part of x_1 that is independent of x_2
- $r_{y(1.2)}$=correlation between salary and GPA with IQ partialled out
- $sr^2 = c/(a + b + c + d)$



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Semipartial correlation (cont.)

- can rearrange semipartial correlation formula:
  - $R^2_{y.12} = r^2_{y2} + r^2_{y(1.2)}$
- R (Multiple correlation) based on information from variable 1 plus additional <u>nonredundant information</u> from variable 2 through variable p so: $R^2_{y.123...p} = r^2_{y1} + r^2_{y(2.1)} + r^2_{y(3.12)} + \ldots + r^2_{y(p.123...p-1)}$
- If the predictors were completely independent of one another, there would be no shared variance to partial out
- would simply sum the r^2 for each X variable with Y to get the Multiple R^2

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Simple linear regression

- Finding the equation for the line of best fit
- Least squares property
  - Minimizes errors of estimation
- The line will have the form: $y' = a + bx$

  Where: y' = predicted value of y
  a = intercept of the line
  b = slope of the line
  x = score of x we are using to predict y
- Multiple regression an extension of this

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Multiple regression models

- Simple linear regression models generally represent an oversimplification of reality
- Usually unreasonable to think in terms of a single cause for any variable of interest
- Our equation for simple linear regression: $y' = a + bx$
- becomes: $y' = a+b_1x_1+b_2x_2+\ldots+b_kx_k$

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Multiple regression (cont.)

- $y'=a+b_1x_1+b_2x_2+\ldots+b_kx_k$
- Where y = dependent variable (what we are predicting)
- $x_1, x_2,\ldots, x_k$ = predictors (independent variables or regressors)
- $b_1, b_2,\ldots, b_k$ = regression coefficients associated with the k predictors
- a = y intercept
- The predictors (x's) may represent interaction terms, cross products, powers, logs or other functions that provide predictive power

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Interpreting b coefficients

- If x_1 is held constant, every unit change in x_2 results in a b_2 unit change in y'
- The constant, a (y intercept) does not necessarily have a meaningful interpretation
  - any time 0 is outside the reasonable range for predictors the intercept will be essentially meaningless

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Overall ANOVA

- Testing overall model adequacy
- Null hypothesis H_0: $b_1=b_2=\ldots=b_k=0$
  Alternative hypoth H_1: any b <> 0
- F statistic (ANOVA) = $MS_{regression}/MS_{error}$
  $MS_{regression} = SS_{regression}/ k$
  $MS_{error} = SS_{error} / n-(k+1)$
- Where n = number of observations
  k = number of parameters in model (excluding a)

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ANOVA (cont.)

- $SS_{regression} = \Sigma(y' - M_y)^2$
- $SS_{error} = \Sigma(y - y')^2$
- When the overall regression model provides no predictive power the two mean square quantities will be about equal
- We are testing an F statistic with (k, n-(k+1)) degrees of freedom

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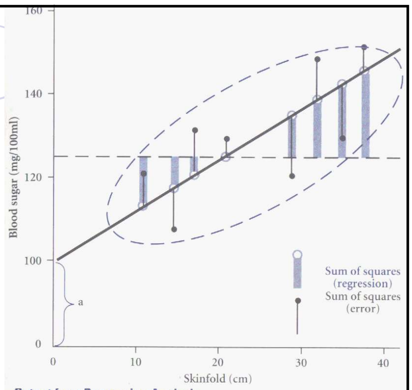
ANOVA in multiple regression



Output from Regression Analysis

Variable	Coefficient	SE	t	Significance
Constant	98.00	12.3	7.97	0.0001
Skinfold	1.14	0.046	24.70	0.0001

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R^2 and Adjusted R^2

- Multiple R^2, coefficient of determination
- Total proportion of variance accounted for by all predictors in the model
- High R^2, values do not necessarily mean that a model will be useful for predicting Y
- Overfitting (adding essentially irrelevant predictors) can result in a high R^2 in the absence of any predictive power
- If we fit a regression model with n-1 predictors R will always be = 1.0 unless we have a rare data set where two cases are identical on all predictors but differ on Y value
- Adjusted R^2 takes into account our n and the number of predictors we have used in the model

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Misuses/problems

- Multicollinearity
  - exists when two or more independent variables (predictors) contribute redundant information to the model
  - can think of conceptually as a problem in assigning unique variance components to variable
  - creates computational problems and coefficients that do not make sense
  - resolve by removing redundant predictor(s)
- Predicting outside the range of data that was used to generate the regression model
  - positive linear relationship
  - relationship may change-resulting in substantial error of estimation
- Failure to explore alternative models
  - need to be familiar with data
  - examine relationships between variables
  - be aware of potential interactions and include in model if necessary
  - try different models

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Standardized betas

- with more than one independent variable:
  - b coefficients cannot be directly compared due to different scales across measures
  - standardized Beta coefficients allow us to compare the relative predictive power of variables in the equation
  - we get standardized Betas by converting all of our predictors to z-scores and running the regression

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Regression diagnostics

- help us assess the validity of our conclusions and their accuracy
- careful screening of the data cannot be overemphasized
- Outliers
  - outliers: data points that lie outside the general linear pattern (midline is the regression line)
  - removal of outliers can dramatically affect the performance of a regression model
  - should be removed if there is reason to believe that other variables not in the model explain why the cases are unusual these cases may need a separate model

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Outliers (cont.)

- outliers may also suggest that additional explanatory variables need to be brought into the model
- multivariate outliers can be difficult if not impossible to identify by visual inspection
- ex. 115 lbs. and 6 feet tall
  - either observation alone is not unusual, but together they are.
- unusual combinations might include odd pairings of scores on separate measures of ability or different indices of production
- multivariate outliers represent results that when considered as a whole do not make sense

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Distance, leverage, & influence

- most common measure of distance (also called discrepancy) is the simple residual
- distance between any point and the regression surface
- identifies outliers on the dependent (y) variable
- leverage statistic, h, also called *hat-value*, identifies cases which may influence the regression model more than others
- with reasonably large n ranges from essentially 0 (no influence on the model, actually the min is 1/n) to 1

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Distance, leverage, & influence (cont.)

- leverage identifies outliers in the dv's
- <.2 fine, >.5 case should be examined
- cases that are high on distance or leverage can strongly influence the regression but do not necessarily do so
- points that are relatively high on both distance and leverage are very likely to have strong influence

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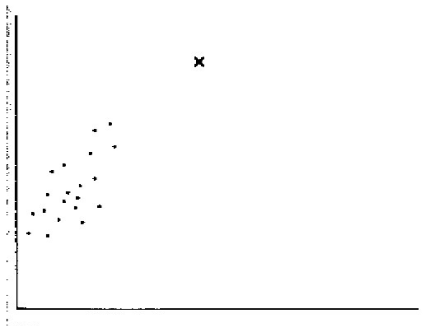
Influence on regression line



(a) High leverage, low discrepancy, moderate influence

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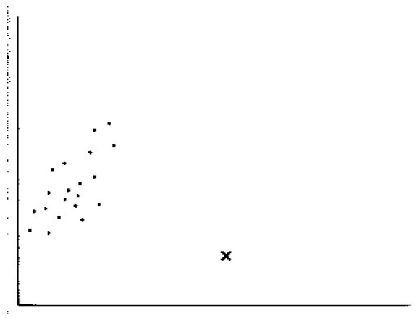
Influence on regression line



(b) High leverage, high discrepancy, high influence

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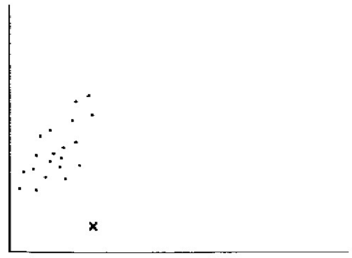
Influence on regression line



(c) Low leverage, high discrepancy, moderate influence

FIGURE 5.1 THE RELATIONSHIPS AMONG LEVERAGE, DISCREPANCY, AND INFLUENCE.

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Cook's distance (D)

- most common measure of influence of a case
- it is a function of the squared change that would occur in y' if the observation were removed from the data
- interested in finding cases with D values much larger than the rest
- cut-off influential cases, D greater than 4/(n - k - 1)
- where n is the number of cases and k is the number of independents
- Cook's distance obtained by issuing "postestimation" command in Stata

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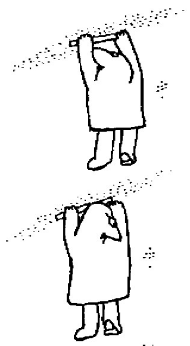
Regression diagnostics



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Diagnostics put to use

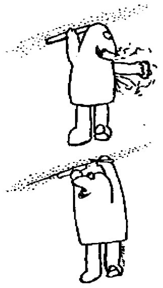

Figure 10.1. Regression diagnostics in action.
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Multicollinearity

- intercorrelation of independent variables
- R^2s near 1 violate assumption of no perfect collinearity
- high R^2s increase the standard error of the beta coefficients and make assessment of the unique role of each independent difficult or impossible
- simple correlations tell something about multicollinearity
- preferred method of assessing multicollinearity is to regress each independent on all the other independent variables in the equation

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Multicollinearity (cont.)

- inspection of the correlation matrix reveals only bivariate multicollinearity, for bivariate correlations > 0.90
- to assess multivariate multicollinearity, one uses tolerance or VIF
- may not be any extremely high bivariate correlations
- if any variable can be represented as a linear combination of other variables in the model, perfect multicollinearity exists

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Tolerance/Variance Inflation Factor (VIF)

- Tolerance
  - 1 - R^2 for the regression of that independent variable on all other independents, ignoring the dependent
  - as many tolerance coefficients as there are independents
  - higher intercorrelation of the independents, the tolerance will approach zero
  - part of the denominator for calculating the confidence limits on the b (partial regression) coefficient
- Variance inflation factor (VIF)
  - reciprocal of tolerance
  - when VIF is high there is high multicollinearity and instability of the b coefficients

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Variance inflation factor (VIF)

R_j	Tolerance	VIF	Impact on SE_b
0.0	1.00	1.00	1.0
0.4	0.84	1.19	1.09
0.6	0.64	1.56	1.25
0.75	0.44	2.25	1.5
0.8	0.36	2.78	1.67
0.87	0.25	4.00	2.0

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